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## Variability in the Demands for Aircraft Spare Parts

### Its Magnitude and Implications

Gordon B. Crawford

January 1988

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40 Years  
1948-1988

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# **Variability in the Demands for Aircraft Spare Parts**

## **Its Magnitude and Implications**

Gordon B. Crawford

January 1988

A Project AIR FORCE report  
prepared for the  
United States Air Force

*40 Years*  
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## PREFACE

For certain recoverable aircraft components, the variability observed in peacetime demands is dramatically greater than assumed by the U.S. Air Force in planning, resourcing, and capability assessment modeling. The system disruptions, resource losses, and inevitable surprises of wartime will greatly compound such variability in wartime.

This work is a major part of the motivating research underlying the set of initiatives called *CLOUT* (*Coupling Logistics to Operations to meet Uncertainty and the Threat*) that has emerged from the Project AIR FORCE project entitled "Enhancing the Integration and Responsiveness of the Logistics Support System to Meet Wartime and Peacetime Uncertainties," popularly known as the "Uncertainty Project." RAND undertook the Uncertainty Project as part of Project AIR FORCE's Resource Management Program under the joint sponsorship of AF/LEX, AF/LEY, and AFLC/XR.

This report is intended for audiences concerned with military logistics requirements and capability assessment. Although it addresses only Air Force data and policies, there are indications that the problems reported here are common to all the services.



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## SUMMARY

Mathematical models of the logistics system are used to determine spares requirements and play an important role in evaluating logistics policies. The kernel of many, if not most, of these models is the modeling of the failure process and the resulting series of random demands on supply and maintenance. This report describes the assumptions of these models, and **quantifies ways in which the behavior of the data differs from the assumptions of the models.** The differences are important and pervasive.

These assumptions about the stochastic behavior of the demands on maintenance and supply that result from operating aircraft permeate the Air Force and drive many of its most elemental policy decisions. They are important to decisions ranging from the collection of spare parts that should accompany a deploying squadron of airplanes to determining the manning of a maintenance squadron or a depot repair center.

Briefly stated, the assumptions are:

1. Aircraft failures are driven by a known operational activity: the expected number of failures of a particular part is proportional to a known and measurable quantity, such as flying hours or landings.
2. The constant of proportionality is known, or can be reliably estimated, and does not change over short time horizons or, in some cases, time horizons as long as five years.
3. The degree of random variation about the mean is known and is adequately modeled in Air Force capability assessment and requirements models.

It is further assumed that the interaction between demands and maintenance is such that these probabilistic assumptions carry over and determine, in known ways, the stochastic behavior of the number of parts in the repair pipeline.

Assumption 1, the so called "Linearity Assumption" has received much attention elsewhere in the literature<sup>1</sup> and is not treated here. The most important results here are that even within the fairly steady state world of peacetime flying activity, **none of the other assumptions above are supported by the relevant data. The**

<sup>1</sup>See Boeing (1970); Casey (1977); Donaldson and Sweetland (1968); Hunsaker et al. (1977); Kern and Drnas (1976); Pederson et al. (1981), Shaw (1981), and IAC (1981).

**discrepancies** between the modeling assumptions about demands and the observed demand data **are both pervasive and important.**

A detailed analysis of time series data, both data on numbers of removals and numbers in the repair pipeline, compares what is observed with what is assumed in the models and attempts to assess the importance of the difference. Additional data are presented in the appendixes, which also contain a discussion of the current approach to modeling the random variation in the demand and repair process. Several new forms of Palm's theorem provide some mathematical justification for common modeling practices and are also included.

Aside from violating common modeling assumptions, the demand variability reported here would not be so important if maintenance was able to keep the numbers in repair pipelines constant and at acceptable levels—it is **pipeline contents, not demand rates, that directly affects aircraft availability.** Unfortunately, examination of the number of parts in the repair pipeline over time reveals even more variability than does the number of demands over time. Not only do repair pipelines exhibit excessive variation about their means, but in the depot portion of the pipeline the means themselves are generally several times larger than the models assume.

These observations have two important ramifications:

1. Excessive demand variability substantially reduces the confidence we can put in our requirements and capability assessment models.
2. Highly variable repair pipelines with means larger than assumed by requirements models have a damaging effect on aircraft availability and wartime readiness.

Depot policies, decisions, and goals should be aimed at reducing these pipelines and increasing aircraft availability and wartime readiness.

## ACKNOWLEDGMENTS

The research in this report was briefed widely within Air Force maintenance and supply organizations. Those audiences had important comments that shaped the form and substance of subsequent research and the report that follows.

The price of gathering the abundance of data used here was paid by supportive Air Force personnel. Then Major David Arnold, Headquarters, TAC, chose the 19 parts studied in detail and arranged for the special data collection. The staff at AFLC/XRS was very helpful; in particular, Barbara Wieland provided additional supply data from the D041 system for a less detailed but wider ranging look at other demand data. James Bias and Phillip Squires, OCALC/MMMA, helped with interpretation of data from the SAFE/D143H system. Lieutenant Colonel Douglas Blazer, AFLMC, shared his results on the variability of EOQ demands.

Special thanks are due to RAND colleague Frederick Finnegan who computerized and massaged the large databases. His recall of who sent what, and what we did with it, provided a thread of continuity to research that began in 1983. James Hodges developed and made clear the statistical aspects of estimating VTMRs. Extracts from his Note on the subject are included in the appendixes. Gus Haggstrom tried to keep us both intellectually honest. John Abell wrote the preface. Hyman Shulman and, especially, Irving Cohen made innumerable important suggestions that gave focus to this research and report. Michael Rich has been the driving force behind getting it all finished, written, and published.

Thanks are also due Raymond Pyles and John Rolph for their comprehensive reviews of a draft of this report, and to Margaret Brackett who cheerfully and capably typed the many versions and corrections.

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## GLOSSARY

AFM	Air Force Manual
AIS	Avionics Intermediate Shop
AWP	Awaiting Parts
CND	Can Not Duplicate
COMO	Combat Oriented Maintenance Organization
COSO	Combat Oriented Supply Organization
D041	Air Force's computerized supply system and database
EOQ	Economical Order Quantity
HUD	Head Up Display
LRU	Line Replaceable Unit
MICAP	Mission Incapable, Awaiting Parts
MSL	Feet above Mean Sea Level
MTBF	Mean Time Between Failures
NFMC	Not Fully Mission Capable
NSN	National Stock Number
NRST'd	determined to be Not Repairable This Station
POS	Peacetime Operating Stock
RETOK	Retest OK (same as CND)
SAFE	Supportability, Analysis, Forecasting, & Evaluation System
SRU	Shop Replaceable Unit
TCTO	Time Compliance Technical Order
VTMR	Variance-To-Mean Ratio
WESEP	Weapon System Evaluation Program
WRM	War Readiness Material
WRSK	War Readiness Supply Kit
WSMIS	Weapon System Management Information System

## I. VARIABILITY IN DEMAND RATE

The first half of this report documents the excessive variability that has been measured in the demand process for aircraft repairable parts. The high degree of variability found in the empirical data is compared with the variability expected by capability assessment and requirements models. There is a wide discrepancy between the degree of variability presumed and present. The implications this disparity has for capability assessment and requirement modeling are then examined.

If every time the demand rate increased it were possible by working faster, say, to keep the pipeline quantities fairly constant, then the demonstrated variability in demand rates might not adversely affect aircraft availability, which is directly driven by pipeline contents. The second half of the report examines successive snapshots of pipeline contents (at both the base and the depot) over time. Unfortunately there is also significant variability there. Not only is the variation in pipeline quantities substantially greater than the models predict, but the mean pipeline quantities observed are also much greater than those predicted by D041 and used in capability assessment and requirement calculations.

Although the demand variability may substantially reduce confidence in requirements and capability assessment models, the pipeline variability, and the unanticipated large pipeline means, may directly decrease aircraft availability.

The supply and maintenance policy of the Air Force assumes a certain degree of self-sufficiency and autonomy within an operating base, even in peacetime.<sup>1</sup> This policy of self-sufficiency is based on several assumptions, including three that are questioned here:

1. There is adequate stock in the system to meet the requirements computed by requirement models;
2. There is sufficient stability in the peacetime demand process that historical averages can be used to accurately predict future peacetime needs; and,
3. There is sufficient stability in the process that historical peacetime demands per flying hour can be used to predict war-time needs.

<sup>1</sup>Although the dependency of air bases on the depots is acknowledged in Air Force stockage models, the bases are presumed to have enough stock to cover the expected base to depot and depot to base pipelines, plus a reasonable amount of random variation, without any unusual management actions.

For many critical parts, there has never been enough stock to fulfill the Air Force's stock policies. Stockage and resupply policies ignore this discrepancy and its probable result—Air Force units may not be able to meet their wartime missions. A reason often cited for overlooking this discrepancy is that the next year's buy will fill these shortages. That has never happened; and in view of the evidence on the variability of demand rates and the commonly acknowledged long procurement lead time for many parts, it is very unlikely that it can ever happen. At reasonable funding levels some parts will always be in critically short supply.

The data raise strong questions about the stability of the peacetime demand process and the ability to predict future peacetime needs adequately on the basis of historical data. In light of the inadequacy of the historical peacetime demand process to predict itself, the assumption that the peacetime demands per flying hour can adequately predict the wartime demand process is even more questionable.

It could be asserted that the differences enumerated are philosophical, that in fact planes do fly, and consequently the effect of these differences between the observed data and the model assumptions is not as great as asserted.

Planes do fly, but standard support procedures at air bases include expeditious workarounds, including WRM withdrawal, expedited base repair, depot priority schemes, lateral resupply, cannibalization, and flying partially mission capable airplanes. We would not claim that these workarounds are a bad management practice in peacetime; the disturbing thing is that most of them won't work in wartime. For instance, WRM withdrawal, a common way of supporting the peacetime fleet, ceases to be a buffer when the partially depleted WRM is all that is available to support a deployment. Expedited base repair will certainly occur in wartime, but depot priority schemes are of little benefit for units cut off from the depot. Lateral resupply may or may not be available. Cannibalization will continue as a way of life, but flying partially mission capable airplanes may be very unattractive in wartime. In other words, the ability to keep the fleet flying in peacetime is no guarantee that the problems enumerated here would not severely curtail or handicap flying in wartime.

These workarounds are ignored<sup>2</sup> in current (or past) requirements models, hence may seem unnecessary, or at the very least atypical. They may, in fact, be routinely necessitated by the differences between real world demand processes and the way we model these processes in our requirements models.

<sup>2</sup>The exception: Cannibalization is considered in D929.

Stockage and capability assessment models assume that removals follow a Poisson type of arrival process, either a simple Poisson or some form of compound Poisson (usually the negative binomial). The models assume that the average rate of removals is proportional to flying intensity, with an unchanging constant of proportionality. Given the mean of the process, the variation around the mean is characterized by a variance-to-mean ratio (VTMR). The variance-to-mean ratio<sup>3</sup> is defined as

$$\text{VTMR} = \frac{(\text{the variance of the number of demands per unit time})}{(\text{the expected number of demands per unit time})}$$

This quantity is a measure of the variability, hence the unpredictability, of the demand processes. The stockage assessment models typically used by the Air Force assume a VTMR between zero and five (or in some cases an expected value calculation is used—essentially ignoring variability and assuming a variance-to-mean ratio of zero). In Stevens and Hill (1973), AFLC/XRS recognized that VTMRs in the data are commonly as big as two and three and may get as big as five.<sup>4</sup> That study resulted in fitting an exponential power curve to a large aggregated database having the historical VTMRs and historical mean demands per quarter for reparable aircraft parts. The curve approximates the VTMR of a part as this exponential function of its average demand rate. D041 uses this curve to predict the VTMR of a part on the basis of its observed mean.

The curves in use begin at zero, climb rapidly to two or three, and are capped at five. To examine the ability of such a power curve to fit and predict the VTMR of a part on the basis of its mean, note Figs. 1 and 2.<sup>5</sup> In Fig. 1 each point corresponds to a part in a 1982 F-15 WRSK. The x-coordinate of the point is the D041 eight quarter worldwide average demands per quarter of the part. The y-coordinate is the VTMR calculated from these eight quarters of data. The curve at the bottom of the graph approximates the power curve used by AFLC to predict VTMRs from means. It may be that roughly half the points lie below the curve and half above it, but neither this curve nor any other can adequately predict future or past observations of VTMRs on the basis of means alone. Figure 2 enlarges the lower left-hand corner of Fig. 1 to examine, in greater detail, the inability of any curve to fit these points. (In both curves, there are no points with very low

<sup>3</sup>Unless otherwise noted, a demand rate per flying hour or per 1000 flying hours is used to compute a "flying-hour-weighted" variance. See App. E or Hodges (1985) for details.

<sup>4</sup>See Hodges (1985).

<sup>5</sup>In Figs. 1 and 2 the VTMR is not flying-hour weighted.

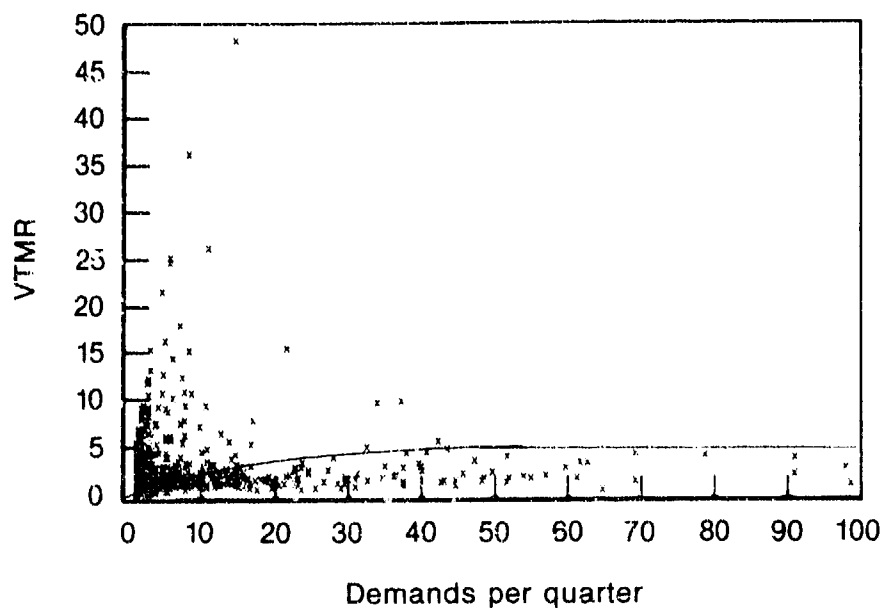


Fig. 1—Scatter diagram: worldwide VTMRs vs. average demands per quarter

demand rates. In this and all subsequent analysis detailing the extent of high VTMRs, items with fewer than five demands per year are discarded. Our intent was to avoid the influence of parts with such low demand rates that their wartime requirements would be easy to satisfy by cannibalization.)

Variance-to-mean ratios are important. Most requirements and capability assessment models assume the flow of broken parts (the "demand process") into the repair facility will be random, but with a certain mean and a certain degree of randomness or "irregularity." The VTMR is a measure of this irregularity. When the VTMR of the demand process for a part is greater than assumed, then the part will arrive at the repair process in what appear to be large random clusters instead of at random, but more evenly spaced, intervals.<sup>6</sup>

The ability of capability assessment models to predict wartime requirements depends on many assumptions other than those about the

<sup>6</sup>Even if an arrival process is stationary Poisson with a VTMR of one, a plot of arrivals over time usually appears to have clusters. If the mean of the process remains constant and the VTMR increases, the apparent size of the clusters appears to increase, as does the length of the intervals between clusters.



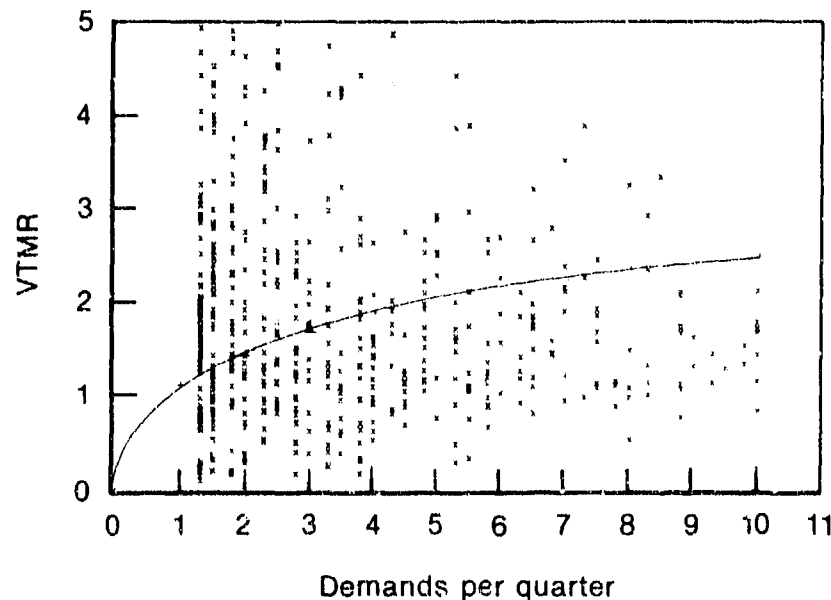


Fig. 2—Scatter diagram: worldwide VTMRs vs. average demands per quarter, large scale

demand process. It is not the intent of this report to address other areas of potential modeling controversy. It is nonetheless reasonable that if failures of a part occur in larger clusters than planned, then the number of a given part in repair, hence the number of shortages of that part at any time, will also have more random variation than planned.

A full cannibalization calculation, as is used in D029 (the Air Force wartime requirements model) and Dyna-METRIC (the Air Force standard capability assessment model) assumes that all "holes" resulting from shortages of usable parts will be consolidated on the minimum number of not fully mission capable (NFMC) aircraft. This means that the number of NFMC aircraft is driven by, and equal to, the number of shortages for the part currently causing the largest number of shortages. The clustering of parts in repair clearly exacerbates this situation: The current "worst part" is very likely to be one that has endured a "cluster" of failures. If clusters of failures are much more likely and larger than planned, the number of NFMC aircraft is also likely to be much larger than planned.

Figure 3 examines the effect of higher VTMRs on capability assessment models and predictions of wartime capability. This analysis has

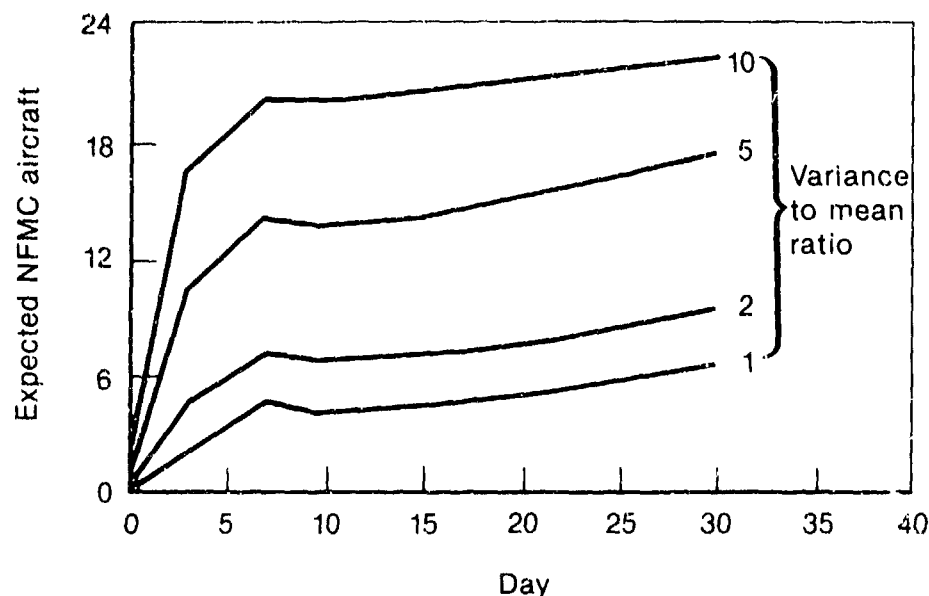


Fig. 3—A gross estimate of the effects of uncertainty on wartime capability

used a D029 WRSK and used the Dyna-METRIC model (Hillestad and Carrillo, 1980; Pyles, 1984) to evaluate the capability of the WRSK to support a squadron of F-15s flying a 30-day scenario at war-like sortie rates.

The issue here is not how well either of these models predicts the performance of a squadron at war. The issue is that our confidence in how well a squadron will do, as a function of the spare parts we send them off with, stems in part from Air Force requirements models. When these requirements models are driven with data that more nearly represent the observed demand streams, the models suggest that the deployed and unsupported squadrons do very poorly.

In the bottom curve, labeled "1," the VTMR of all parts has been set equal to 1, as is assumed in D029. On day 30 the number of expected not fully mission capable (NFMCAircraft) in this calculation has risen to approximately 5, the same number that is set as a target in D029. In this case, the Dyna-METRIC calculation compares well with the D029 calculation that was used to construct this kit. If, however, the VTMR of those parts that are driving the NFMCAircraft is assumed to be 2, then the number of NFMCAircraft on day 30 is approximately 9 instead of 5. Data presented below show that the VTMRs for important and critical parts are often in excess of 5. If the driving

parts in this WRSK calculation, which is a full cannibalization calculation, have a VTMR of 5, then on day 30 there will be 16 (not 5) airplanes NFMC out of a squadron of 24. Even more troublesome: The number of NFMC airplanes rises to over 12 in the first week. Thus, in capability assessment models, the VTMR assumed for the demand process is very important.

These results, and our capability assessment and requirements models, ignore such actions as working faster to enhance productivity and repair of needed components in wartime. To the extent that these options can be understood and quantified, they too should be modeled.

## II. THE 19 PARTS DATA

The most detailed data is a collection examined for this report called the "19 parts data", with which it is possible to examine in detail the demand process by base for each of five TAC bases for select F-15 parts. Since the 19 parts may not be representative of parts in general, the analysis below compares the statistical properties of the 19 parts data with D041 data for all F-15 parts. The results of this comparison are not very reassuring.

In 1980 Headquarters TAC was asked to identify approximately 20 parts that had been in short supply and important MICAP causers as of mid-1980. TAC/LGS responded with dumps from the base level 1050 computers giving demand data by quarter for each of the five TAC bases having F-15s. The data included 16 LRUs and three SRUs of the F-15 aircraft. Some were actuators, some were engine parts, some were expensive, and some were cheap. The items were purposely chosen to be fairly representative of the spectrum of repairable parts on the plane. With TAC's help demands were collected by quarter by base for all TAC bases for these 19 parts—10 to 12 quarters of data beginning in mid-1980 were collected from Langley, Luke, Holloman, Eglin, Nellis, and Hill Air Force Bases. The LRUs that make up this sample are all wartime mission critical. (Unless otherwise noted, the analysis that follows has omitted the data from Luke, a training base, as their operating procedures are unique.)

Most of the parts had strikingly large VTMRs. Commonly accepted theory<sup>1</sup> suggests that demands should be approximately Poisson distributed and VTMRs should be near 1, but about half of the parts had VTMRs of 5 or more. When a VTMR is being calculated from empirical data, a certain amount of deviation away from 1 is to be expected as a result of random variation in the sample.

Because it is a ratio, the VTMR's precise distribution is difficult to compute analytically for small samples. For large samples the distribution of the VTMR is approximately  $\chi$ -square. To avoid the possibility of being misled by large sample theory the distribution of small sample VTMRs was computed with a Monte Carlo program. See Fig. 4.

<sup>1</sup>Although difficult to reconcile with the physical attributes of the demand process, AFLC (Stevens and Hill, 1973) has recognized that using VTMRs greater than one in D041 improves the fit of the modeled demand process to observed real world processes. Although helpful, this procedure falls short of adequately modeling the observed demand processes. This is discussed in more detail below.

With ten data points and the mean anywhere from 1 to 300, an observed VTMR larger than 1.5 makes questionable the hypotheses that VTMRs = 1. A VTMR larger than 2 or 3 is incompatible with the notion of a simple Poisson arrival process. About half the parts exhibited VTMRs larger than 5. Why?

With computer printouts showing two years of supply data detailing the high degree of variability in the demand rates, I visited Luke and Holloman AFB to meet with maintenance chiefs and flight line personnel. Together we reviewed the demand history from the local base for each part, and the maintenance personnel recounted their explanations for the peaks in the demand rate. Some explanations were general in nature and are given below. Others were part-specific and are given in the part-by-part discussions below and in App. A.

Sometimes the explanations were the same at the two bases, but generally the bases were very different. Luke and Holloman both flew about 5000 hours per quarter, but Luke was a training base and had about a third more airplanes. Being a training base, Luke did not have a WRSK and gets less priority for needed parts from AFLC.<sup>2</sup> The

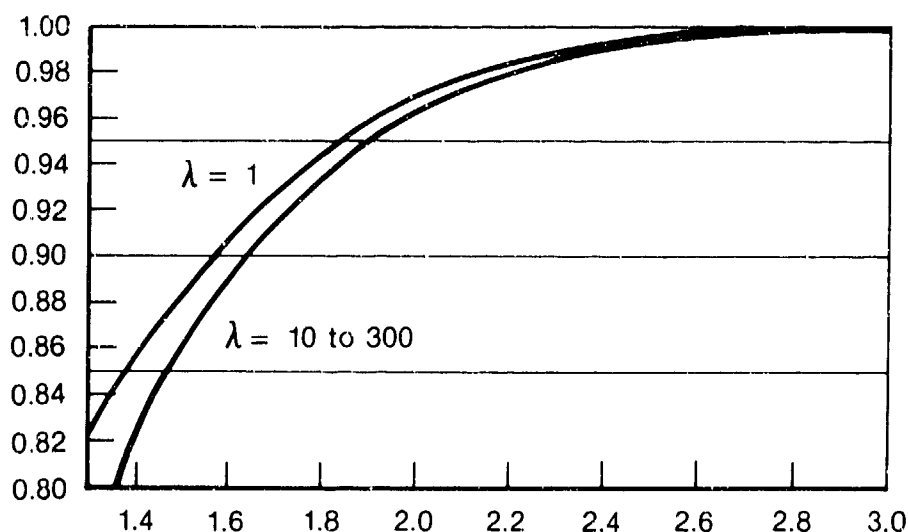


Fig. 4--Distribution of the variance-to-mean ratio

<sup>2</sup>The effect of this on demand variability is not clear, but in some cases, it may result in Luke's not getting high demand parts until they are in ample supply, hence shipped from AFLC in multiple lots. The receipt of multiple lots may result in the near-simultaneous recording of multiple demands.

implementation of COSO (Combat Oriented Supply Organization) also had a surprisingly large effect on the recorded demand rate for some parts.

COSO and its companion COMO (Combat Oriented Maintenance Organization) were Headquarters TAC initiatives to encourage the use, in peacetime, of procedures that are expected to be commonplace in wartime. These procedures involve increased reliance on remove-and-replace actions instead of remove-repair-and-replace maintenance actions, and they require close attention to the AFM 67-1 documentation procedures. As a result, although the true demand stream may not have changed at all, recorded demands have increased for several components during the quarter that COSO and COMO were implemented. (COSO and COMO were implemented on 1 March 1982 at Luke and on 1 February 1982 at Holloman.)

Summer weather at Luke (1090 ft MSL) can be excruciating. Night time temperatures may drop below 60°F. During the day, temperatures on the ramp often reach 120°F. I was told that under these conditions temperatures in the avionics bay of idle aircraft may exceed the Mil Specs for solid state devices. During aircraft start up, power is applied to the avionics before the environmental control system becomes effective. The day to night extremes in temperatures were also cited as contributing to leakage in hydraulic actuators.

At Holloman (4093 ft MSL) daytime temperatures are much milder in the summer, but the weather was also cited as contributing to failures in the third quarter of the calendar year.

The occasional need to deploy aircraft for training was also cited as causing increases in the demands for certain parts. During the second quarter of 1982, Holloman deployed 16 aircraft to the Red Flag exercises at Nellis and seven aircraft to the WESEP exercises. Such deployments require that all participating aircraft have all their offensive systems (and HUD cameras for scorekeeping) fully operational. This may induce component removals as systems are tuned for optimal performance.

To a certain extent demand patterns may follow parts availability. In other words, if a component has a suspected or known malfunction, but no replacement parts are available, the plane may continue to fly with the defective component until supply can provide a replacement. If supply gets a shipment of several parts, a cluster of removals will follow. This explanation for clustering of demands was cited several times at Luke.

Figure 5 indicates heat exchanger demands per thousand flying hours. The F-15 has two heat exchangers, which work independently. They cool lubricating oil and hydraulic fluid by transferring heat to

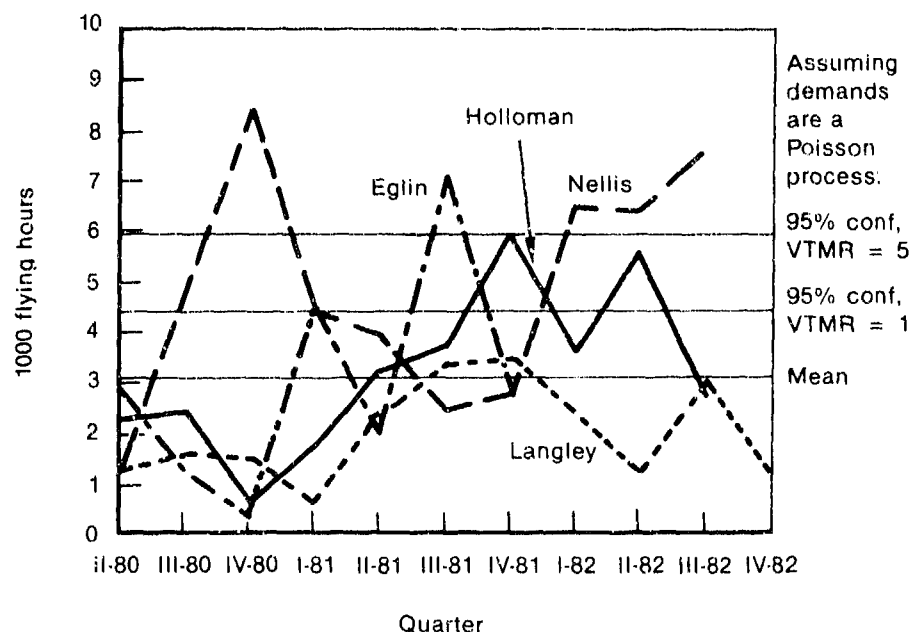


Fig. 5—Heat exchanger demands per 1000 flying hours  
 (average VTMR = 3.5)

fuel that is pumped from the center tank through the heat exchangers to the wing tanks to cool. A common mode of failure is for the sensing elements not to work in consort, resulting in one wing being heavier than the other as one heat exchanger pumps more fuel. The resulting wing imbalance severely restricts the aircraft's flight envelope. At Luke, warm summer weather increases the amount of fuel that gets pumped into the wing and aggravates discrepancies in the sensing elements. At Holloman, in 1981 the depot went to a new thermal sensing element, and the problem actually got worse. Further, an aircraft modification resulted in a reduction of the tolerable wing imbalance, thus increasing the demands for heat exchangers.

At any base, the average demand rate<sup>3</sup> shows wild swings from quarter to quarter in the number of heat exchangers used, swings that are far in excess of what could be explained by the simple Poisson model with a VTMR of 1. The demand rates also seem to differ markedly from base to base, and the swings have little, if any, apparent

<sup>3</sup>All of these plots have compensated for flying hours and graphed the demands per 1000 flying hours.

correlation from one base to another. In short, these data sources contain very little information that would allow predicting, with any confidence, the number of heat exchangers that would be demanded at any given base at any given future quarter, much less in a wartime situation some years into the future.

The identified potential contributors to a changing demand rate for heat exchangers were: weather, an LRU modification intended to decrease the demand rate that seemed to have the opposite effect, and an aircraft modification. Having this hindsight may make the past demand history more understandable, but these factors cannot be of much help in planning for a deployment to an unknown location at an unknown time in the future.

Figure 6 plots the observed demands per thousand flying hours for the converter programmer, an electronic device composed of digital and analog electronics. It functions as an interface between the pilot and the fire control system and between the pilot and the air-to-air missiles. Without the converter programmer the pilot of the F-15 air superiority fighter has no offensive or defensive capability unless he is fortunate enough to achieve and hold a lock on by "bore sighting" his opponent and using his gun or heat seeking missile. Again, there are

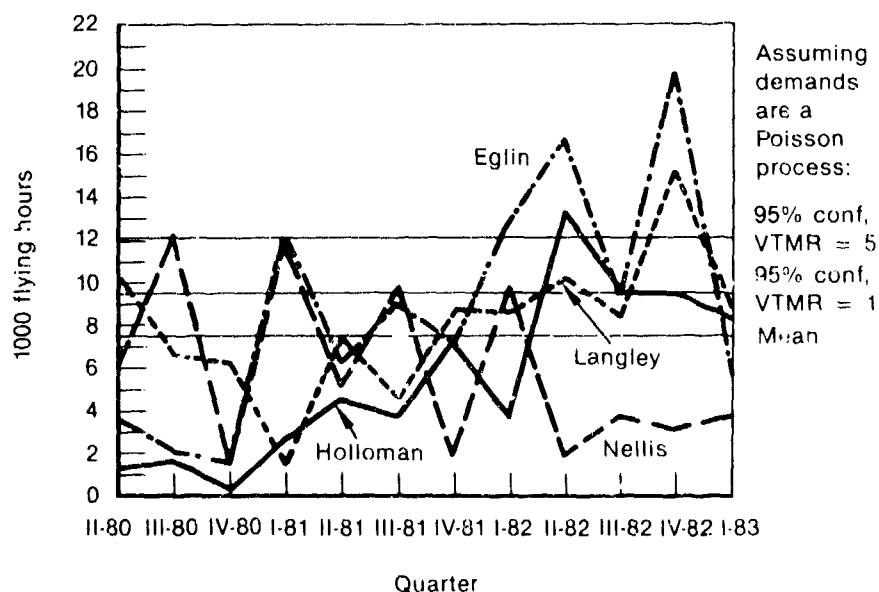


Fig. 6—Converter programmer demands per 1000 flying hours (average VTMR = 9.0)



wild swings from quarter to quarter in the demands for any given base and substantial between-base discrepancies. The observed VTMR at Luke was about three. Most striking in the data is the jump in demand rates at Holloman beginning in the first quarter of 1982. Two factors were cited as contributing to that jump: The first was that COSO began in February of that year; the other was a software change on the AIS that has apparently increased its ability to trouble shoot the LRU, thereby reducing the number of CNDs.<sup>4</sup> Even more disconcerting than the swings in the demand rate, at Bitburg AFB the average demand rate for the converter programmer was about eight times greater than observed at TAC bases. In Europe, common practice is to test the converter programmer with an uploaded radar missile, a more exacting test of the LRU than in TAC where infrared missiles are primarily used. Unfortunately, TAC data dominate the D041 database, not the Bitburg data, which may be far more relevant to wartime.

Figure 7 treats the main landing gear wheel, an item that has been, and continues to be, very troublesome in the F-15. It has a high demand rate and an extraordinarily high observed VTMR on the order of 50 or more. The same pattern persists. With data like these, accurate predictions of the demands that will occur at any base during any future quarter become impossible.

Figure 8 looks at the unified fuel control. The remarks above hold here as well. A glance through Appendix A will assure the reader that the parts selected and highlighted here in the text are typical of the complete sample of 19 parts.

<sup>4</sup>Can Not Duplicate—meaning that the deficiency purportedly observed in the aircraft cannot be duplicated in the backshop (sometimes referred to as RETOK - RETest OK).

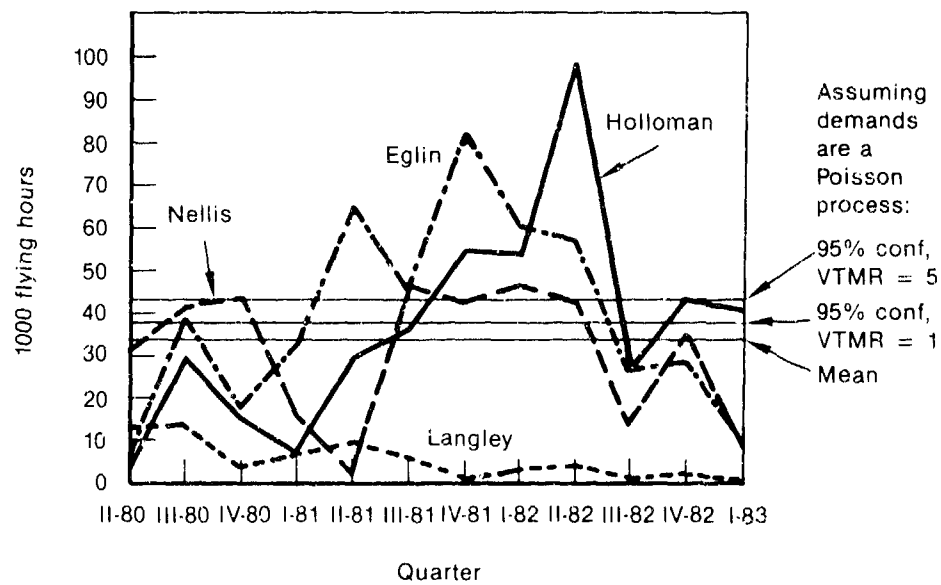


Fig. 7--Main landing gear wheel demands per 1000 flying hours  
(average VTMR = 51.9)

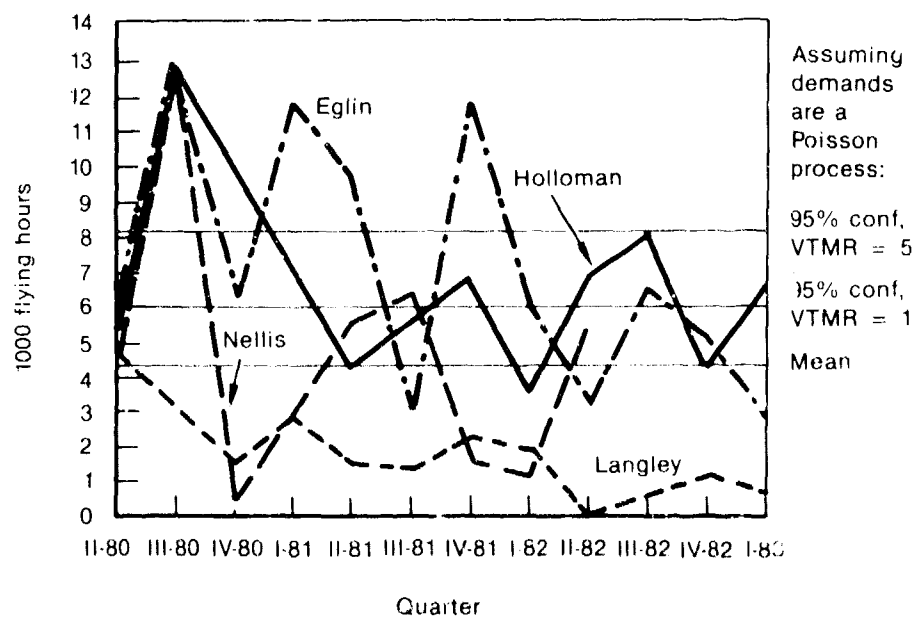


Fig. 8--Unified fuel control demands per 1000 flying hours  
(average VTMR = 3.9)

### III. VARIANCE-TO-MEAN RATIOS: ALL F-15 PARTS

The 19 parts and five bases yield calculated VTMRs for 95 part-base combinations. The distribution of these 95 VTMRs is shown in Fig. 9. Only 25 percent of these VTMRs are less than two, 50 percent of them are less than four and one-half, and 20 percent of them are greater than eight. Recall from Fig. 3 that on any day of the war after the first week, if the driving parts in a WRSK kit have a VTMR of five or greater, the expected number of FMC airplanes is a small fraction of what is expected—and needed.

To avoid being misled by 19 parts that may be atypical, Fig. 10 gives the VTMR (calculated around the base mean) for about 800 F-15 parts using one year's worth of data from a 1981 D041. They appear far more reasonable than do the 19 parts. Only 10 percent have an observed VTMR greater than 5. Although this is reassuring, if the 10

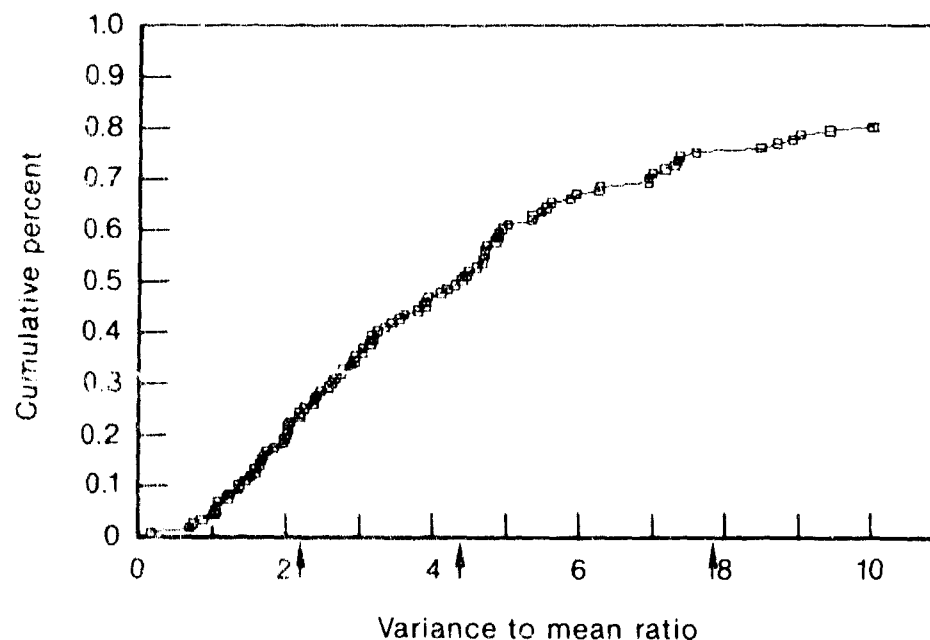


Fig. 9 --Cumulative base level VTMRs, 19 parts data

percent with high VTMRs are those that drive the capability of a squadron (recall Fig. 3), then there are apt to be 18 aircraft NFMC, not five (the target in DO29). Moreover, the results of Fig. 10 are much too optimistic: It shows the variation around the base level mean when in fact DO29 and D041 calculate requirements on the basis of a worldwide mean.

Figure 11 recomputes the variation about the worldwide mean, and plots the calculated VTMRs. In this case 25 percent of all F-15 parts exhibit a VTMR larger than 5.

Although the 19 parts that TAC identified seem to be worse than parts in general, observations of high VTMRs are nonetheless very prevalent in worldwide and base level data. As mentioned, TAC/LGS chose the parts because they were problem parts causing a lot of MICAP incidents in mid-1980. It is not certain whether the parts identified by headquarters TAC were problem parts because they had high VTMRs or vice versa, but the high degree of variability exhibited by those parts is not unusual.<sup>1</sup>

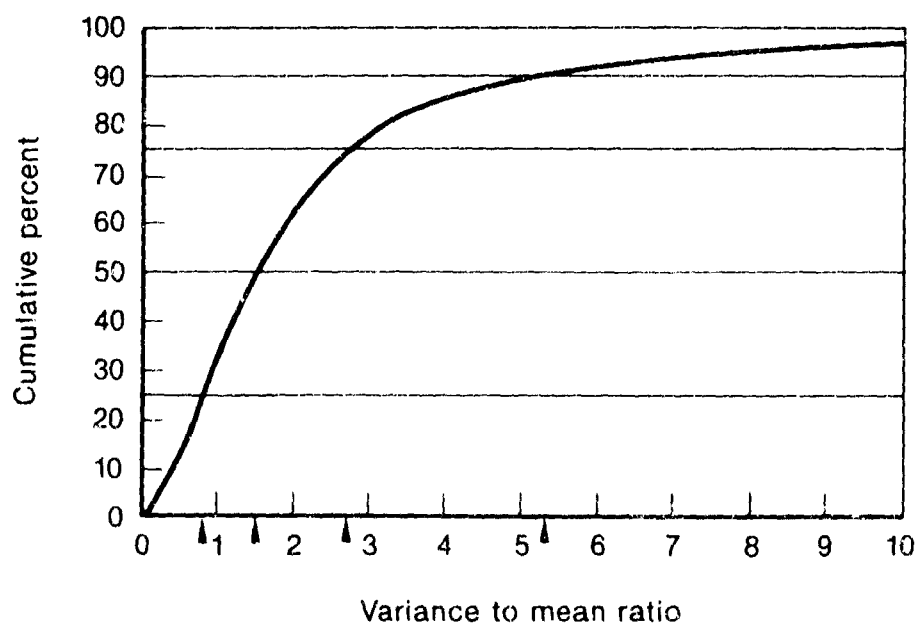


Fig. 10—Cumulative base level VTMRs, all parts

<sup>1</sup>Although the data are not presented here, the VTMRs for the number of items NRST'd at a base have been calculated from empirical data. These numbers were

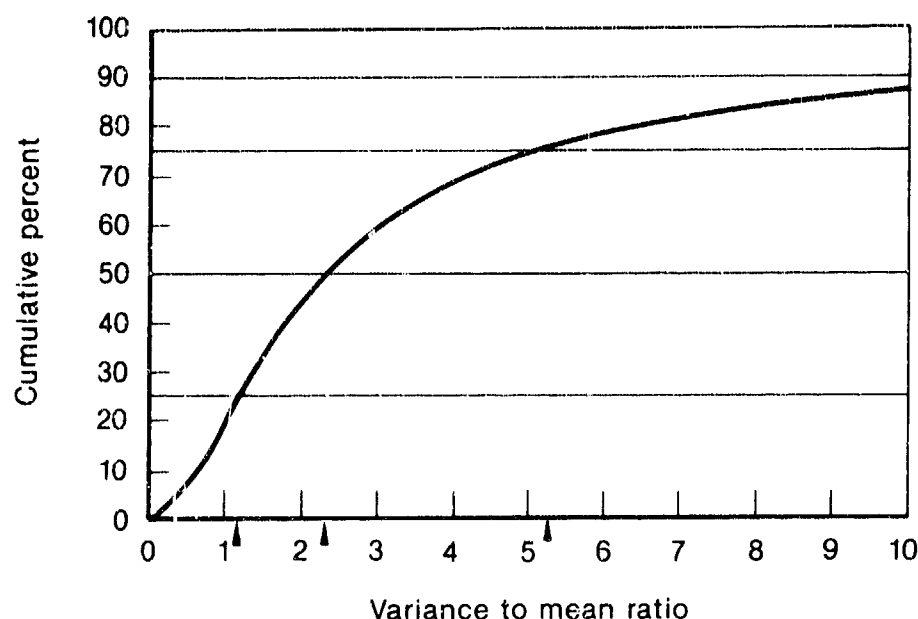


Fig. 11—Cumulative VTMRs, all parts, worldwide means

Although this analysis concentrates on VTMRs, they are not the problem. Variance-to-mean ratios are a way of measuring the problem and are symptomatic. The problem may be that parts have highly variable demand rates, but there are indications<sup>2</sup> that the mean demand rate for many parts shifts over time.

To quantify the extent to which high degrees of variation are measured about a fixed mean, or a mean that itself varies, requires some detailed semantic distinctions that are difficult to justify with the available data. Regardless, logistics models that assume constant means and Poisson arrival processes do a poor job of modeling real-world demands for real-world airplane parts. One message the data have exhibited is that modeling capability assessment or computing requirements is very difficult.

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slightly lower than the observed VTMRs for demands, suggesting that peaks, and perhaps valleys, in the demand process may be "managed" to some extent by maintenance. Several instances of "managed" demands were cited at Luke and Hooloman that contributed to peaks in the demand process and high VTMRs *but had no effect on flying schedules*. These demands could have been similarly managed, or avoided, in wartime.

<sup>2</sup>Calculated VTMRs seem to increase as the observation period gets longer. In addition, where the period of observation is long enough, demand rates for many parts appear to have definite jumps or trends over time.

Most requirements and capability assessment models assume that the amount of time a part spends in the repair pipeline is independent of the demand process (see App. F). In practice this assumed independence is rarely to be expected. If the repair process requires queuing for a piece of test equipment or skilled personnel, or if the repair requires subassemblies that may be unavailable, then the demand process and the repair times (including AWP time) are apt to be positively correlated—during periods of high demand parts are likely to take longer to repair. If there is little or no queuing, and repair parts are available, maintenance personnel will probably do whatever is possible to induce a negative correlation between the demand process and the repair times, so that during periods of heavy demand the repair times are shortened whenever possible.

If a maintenance-induced negative correlation dominated whatever queuing occurs, it would make the pipeline contents less variable than the demand process. As shown in App. F, under the independence assumption that is common in the models, the repair pipeline contents will also have a VTMR near, or less than, that of the demand process.

Thus one important reason for looking at the VTMR of pipeline contents is to answer the question, "Does the repair process mitigate the high VTMRs of the demand process and thereby redeem the models that predict the wartime capability of a deployed squadron?"

There is another reason for looking closely at pipeline variability: An excessive number of parts in the repair pipeline, not an excessive number of demands, causes aircraft to be NFMC for lack of parts. Regardless of the way repair pipelines are modeled, their contents and their variability are important to aircraft availability.

#### IV. VARIABILITY AND PREDICTABILITY OF REPAIR PIPELINES

Section III demonstrated that highly variable demand rates are common; VTMRs greater than 5 are not unusual. At Luke and Hooloman AFBs I met with maintenance crew chiefs in an attempt to understand the causes of these variations. The peaks in variability do not seem to be related to, or predictable from, elements of the standard Air Force data systems. In short, current modelling of the demand process for repairable parts is inadequate.<sup>1</sup>

Aircraft availability is directly associated with and dependent on pipeline contents, not demand rates. In other words, if every time demand rates jumped up, maintenance was able to repair parts at an accordingly faster rate, the pipeline contents would remain fairly constant, and aircraft availability would remain approximately the same as before the increase in demands. That is the reason for investigating pipeline contents, their stability over time, and our ability to predict them.

Requirements and capability assessment models often assume the demand process has a VTMR of one. Although this is explicit in the documentation of some models and implicit in others, the operative assumption in the design and coding of the commonly used models pertains to the VTMR of the number of parts in the repair pipeline, not the VTMR of the demand process. In models that have come to our attention that assume a VTMR of one for the demand process, or allow the user to set his own choice of VTMR, or compute a VTMR other than one, the VTMR is used to describe the VTMR of the number of parts in the repair pipeline, not the input process per se. Even so, users typically estimate the VTMR of the demand processes and use those estimates for the VTMR input.

Appendixes E and F give new statements and proofs of conditions that insure that the VTMR of the demand process is equal to, or an upper bound for, the VTMR of the number of parts in the repair pipeline. All of these conditions require the "infinite server" assumption, which implies that there is no queuing for repair. If there is queuing, and especially if the queuing process is operating near saturation, the VTMR of the repair process may be substantially greater than that of the arrival process.

<sup>1</sup>Unfortunately, work at the Logistics Management Center (Blazer, 1984) indicates that this problem is not limited to repairable parts: EOQ variability appears even higher.

For these reasons, as well as the reasons given at the end of the last section, it is important to look at the variability of the pipeline contents. The following analysis uses data from the D143H system that has been collected in the Supportability, Analysis, Forecasting and Evaluation (SAFE) system at OCALC/MMA. With the help of the MMA staff D143H tapes were collected from all depots. These tapes provide a snapshot of the pipeline contents at the end of every month, beginning in February of 1983. The analysis discussed here includes data from February 1983 to November 1983. The pipeline data for F-15 parts follows. Appendixes B and C have the corresponding data for the F-16 and C-5.

The depot pipeline for unserviceable parts is defined to include those parts in retrograde shipping from the base to the depot or from the depot to a contractor, as well as parts that are unserviceable or in repair at the depot. The base pipeline is simply those parts in repair at the base. Serviceable parts enroute to the base have been classified as serviceable at the base.

The following analysis divides the parts into classes. Recall from Sec. III that the VTMRs calculated from the 19 part data were statistically greater<sup>2</sup> than the VTMRs calculated from all parts. Four classes of assets are defined<sup>3</sup> for analysis: The first class is ALL—all NSNs having one or more in the repair pipeline as of the end of any month, February through October of 1984. The next class is called the BAD 200. These were the worst 200 of the above assets when judged by the ratio of the average serviceable number of assets during the time period divided by the sum of the POS plus WRM requirement worldwide. The third class of assets (the BAD 100) is the worst 100 judged by this same criterion. The fourth class of assets (MICAP) is the worst 50 when judged by the criterion of total number of MICAP hours from February through October 1984.

Figures 12 and 13 show the total number of parts in each of these classes and their value. The class "All" comprised approximately 650 NSNs and 115,000 parts, worth about \$3.8 billion. The BAD 200, when filtered through data checks, yielded reliable information on 144 NSNs and a total of about 10,000 parts worth about \$800 million. Similarly, the BAD 100 yielded 57 parts worth approximately \$20 million. The MICAP 50 yielded 23 NSNs, also worth about \$20 million.

<sup>2</sup>If the cumulative distribution functions (cdf) of the two samples of VTMRs were plotted on the same graph, the cdf of the 19 parts sample would lie below the band to the right of the cdf of the "all parts" sample.

<sup>3</sup>Although it would have been useful to rank parts according to their direct contribution to available aircraft, the difficulty of obtaining such a ranking led to the procedure used here.



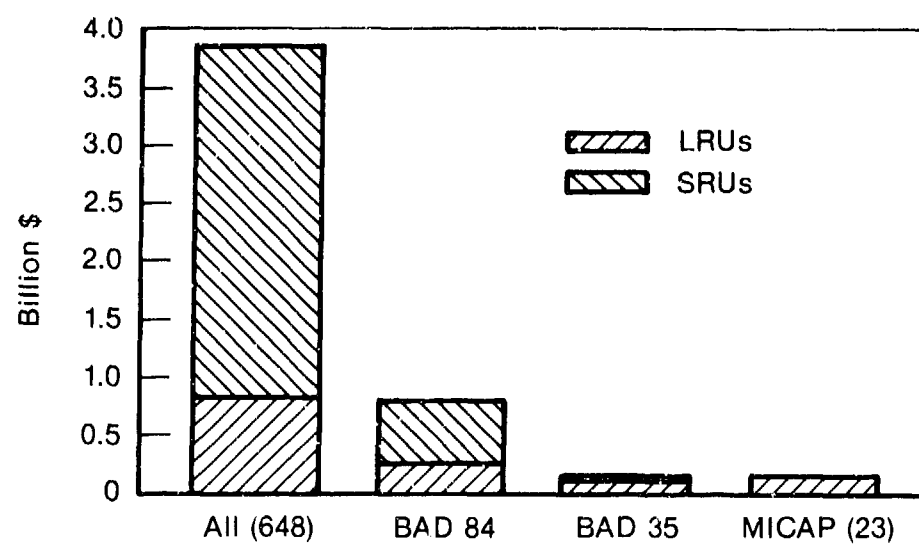


Fig. 12—Cost of F-15-peculiar parts in sample, by class

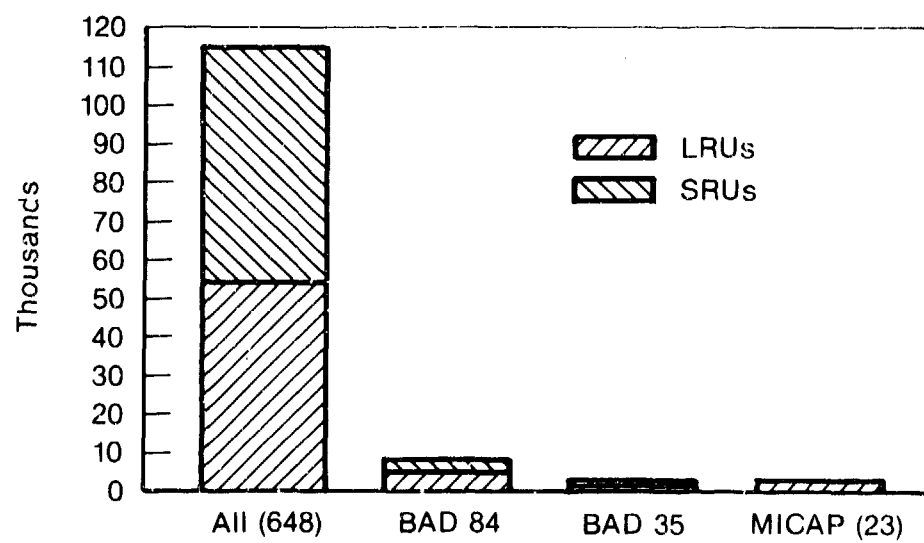


Fig. 13—Number of F-15-peculiar parts in sample, by class

Figure 14 shows the distribution of the observed pipeline VTMRs for all F-15-peculiar assets. The observed VTMRs at the base are statistically lower than the observed VTMRs at the depot, but the depot has many more parts and hence dominates the combined pipeline. (The depot pipeline plus base pipeline —BOTH—lies very close to the graph for the depot only.) In this example, 25 percent of all NSNs had pipeline contents whose VTMR was greater than 6, actually worse than the data for worldwide F-15 demands in general<sup>4</sup> and comparable to the data for F-15 demands computed around worldwide means.

Looking at the smaller classes of parts that are typically more important, the situation seems to get worse, not better. Among the BAD 144 (Fig. 15) fully 25 percent of the parts have VTMRs larger than 10 when the depot or the combined depot and base pipeline are considered. In the BAD 57 (Fig. 16) the situation is unchanged. For the MICAP parts in Fig. 17 the situation is worse yet. Even more parts have observed VTMRs larger than 10. The median VTMR for MICAP parts is about 4.

These analyses have actually understated the sample VTMR of the pipeline contents. The pipelines are dominated by depot pipelines, and most parts are in the depot pipeline for several months or more. The

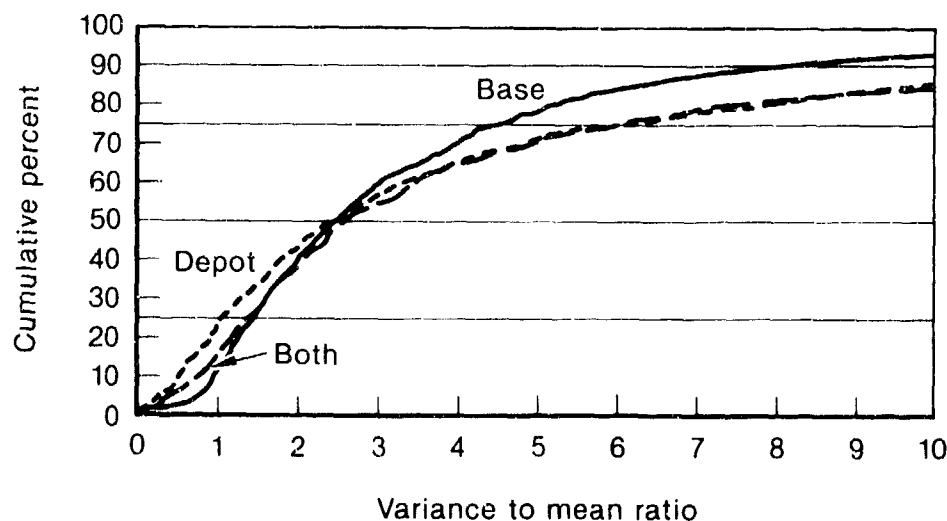


Fig. 14—Cumulative F-15 VTMRs, all peculiar and common parts (1039)

<sup>4</sup>In fact the data were from different time periods, but other observations not reported here suggest that the underlying distributions of VTMRs are fairly stable.

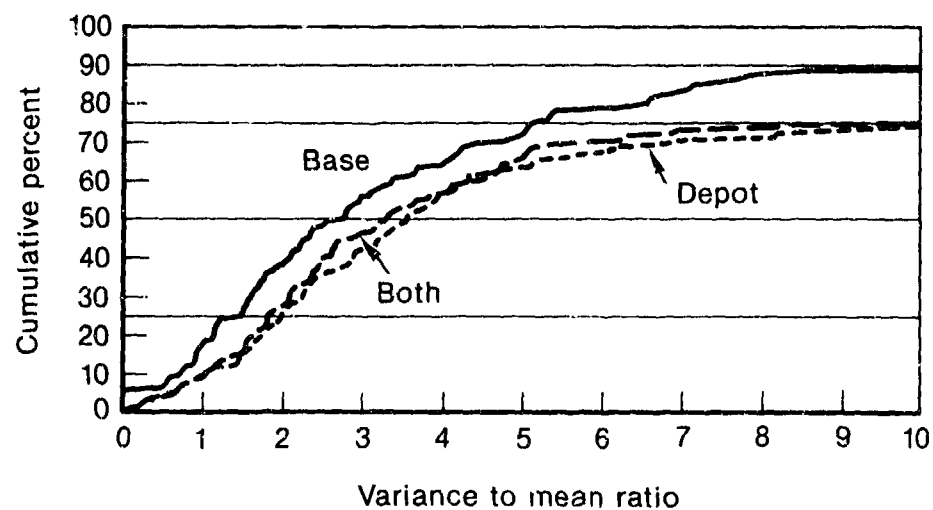


Fig. 15—Cumulative VTMRs, F-15 BAD 144,  
peculiar and common parts

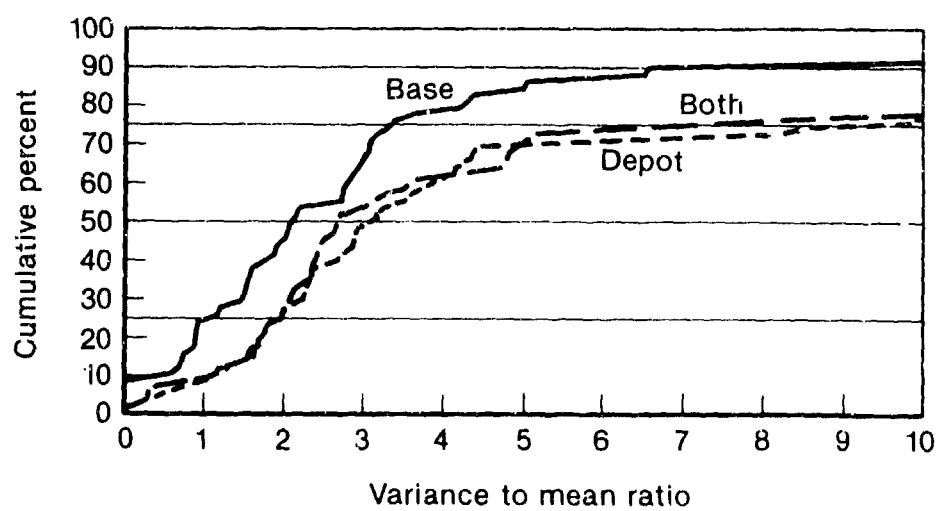


Fig. 16—Cumulative VTMRs, F-15 BAD 57,  
peculiar and common parts

data used here have been monthly snapshots and are positively correlated from month to month. Thus the VTMRs exhibited in these graphs are probably less than the VTMRs that would be observed from an equal number of samples taken at a wider spacing, for instance every three or six months instead of every month. In other words, the positive correlation actually diminishes the measured VTMR.

Further, the variation in pipeline contents around the observed pipeline means have been computed. In the case of the base level pipeline, the mean can be expected to be approximately equal to the observed mean, because the base level mean pipeline contents as given by D041 are largely based on empirical data.

Unfortunately, the expected pipeline contents for the depot pipeline as given by D041 are dominated not by observed values, but by nominal values that have been set by AFLC personnel; hence there is an important second question: How do the observed pipeline means compare with the D041 preset pipeline means? (For convenience, these preset values are referred to as the expected values.) For ALL parts note Fig. 18. (Figures 19 through 21 give comparable data for the other classes of assets.) At the depot during this time period, there were approximately 2.5 to 3 times more parts than D041 expects. Thus, not only are pipeline contents extremely variable about their mean, but their means are approximately 2.5 to 3 times the number that is assumed when computing requirements or predicting capability. At the

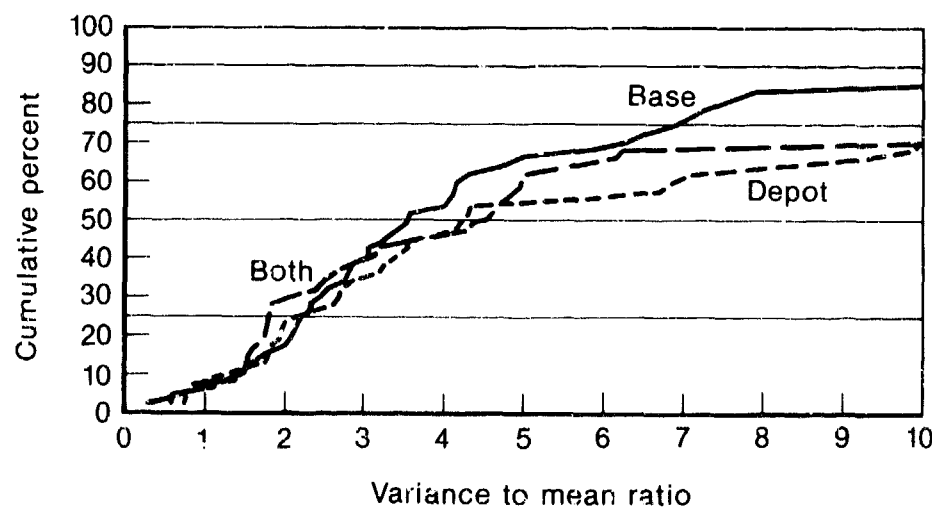


Fig. 17—Cumulative VTMRs, MICAP (46), peculiar and common parts

base, the picture is somewhat better. The observed quantity is only slightly greater than predicted. This is to be expected: The D041 expected base pipeline is based primarily on observed quantities, but the calculation excludes assemblies that are AWP. However, assemblies that are AWP are counted in the observed pipeline.

Figure 18 also gives the relative costs of pipelines—observed vs. expected, base and depot. Again, the depot, in terms of cost, has 2.5 to 3 times more unserviceable parts than the number D041 predicts. At the base level the observed is only slightly higher. Going on to the other classes of assets, the picture remains basically the same. The number of assets in the depot pipeline is still 2.5 to 3 times greater than D041 predicts. Regarding the MICAP assets, one would hope that these assets, which are given high visibility and much priority attention at the depot, would present a much better picture. Yet for these assets there are approximately twice as many unserviceable at the depot as predicted by D041 (Fig. 21). This picture is especially disconcerting because even giving these assets priority treatment, the observed pipeline contents are still twice as big as the expected pipeline contents computed from D041 factors. Yet these factors, which are obviously very wrong, are currently the basis for all capability assessment and requirements modeling.

Figures 22 through 25 provide a snapshot of how serviceable and unserviceable assets are distributed through the system. Looking at the class of all assets, in terms of numbers, approximately 60 percent are serviceable, 3 percent are unserviceable at the base, and 36 percent are unserviceable at the depot. In terms of cost, the picture changes somewhat. The cost of the assets that are unserviceable at the base is somewhat higher, approximately 13 percent of the total. This reflects the decision to make the more expensive assets base repairable. Looking at the more critical assets in Fig. 23, the number of serviceable LRUs drops from 60 percent to approximately 30 percent, and the number of assets unserviceable at the depot has risen from 37 percent to 60 percent. Among these more critical assets, substantially more are unserviceable at the base also, rising from 3 percent to 10 percent. Looking at the cost of assets and how they are distributed, the change is as before: Unserviceable at the base becomes 23 percent, reflecting the higher cost of these parts. Going on to Fig. 24, the BAD 57 assets, the situation is much the same. Serviceable assets are again about 30 percent, and 66 percent are unserviceable at the depot. Among MICAP assets—those expected to get prompt attention—the number that are unserviceable at the base drops to about 6 percent, but still fully 60 percent of the items that cause MICAP incidents are unserviceable at the depot.

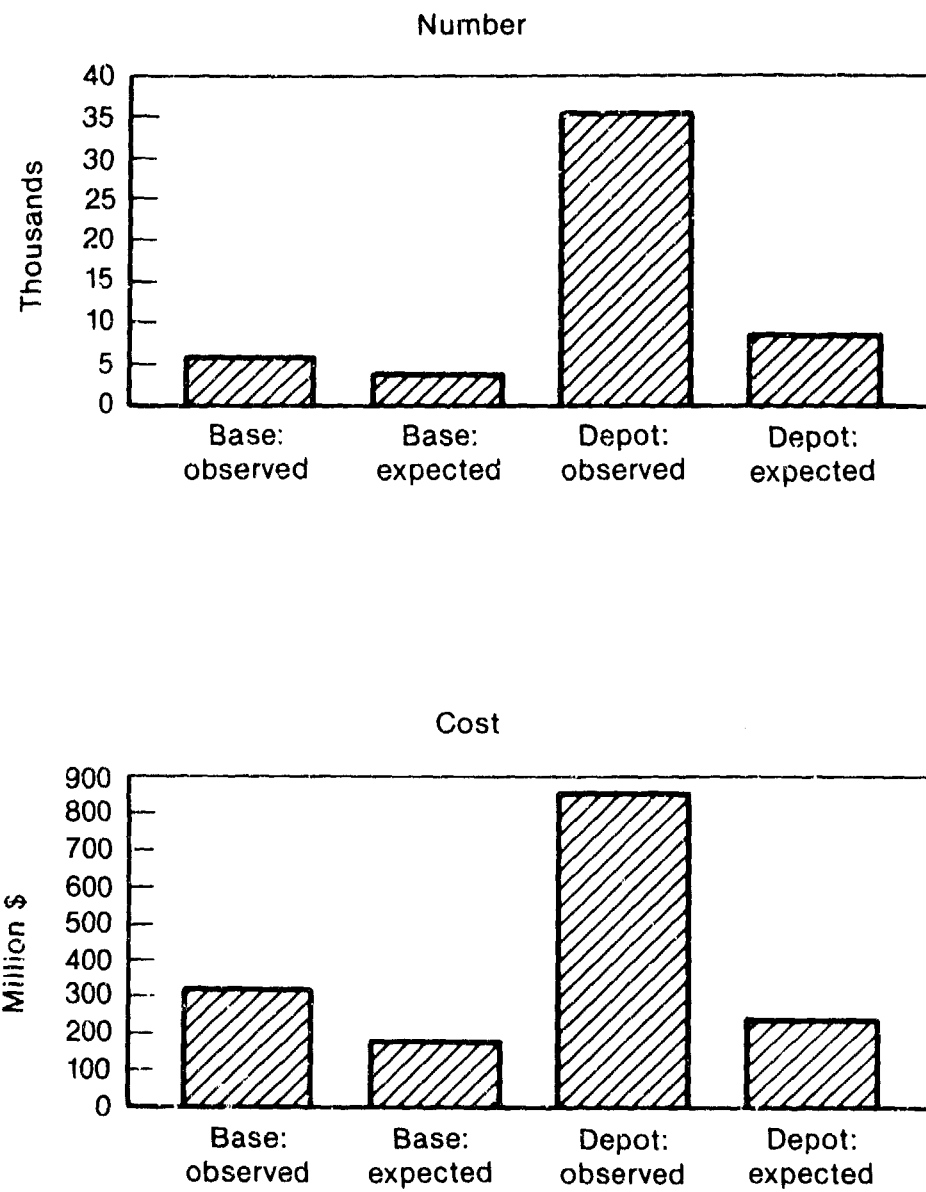


Fig. 18—F-15: Number in and cost of LRU pipeline,  
all (648) peculiar parts

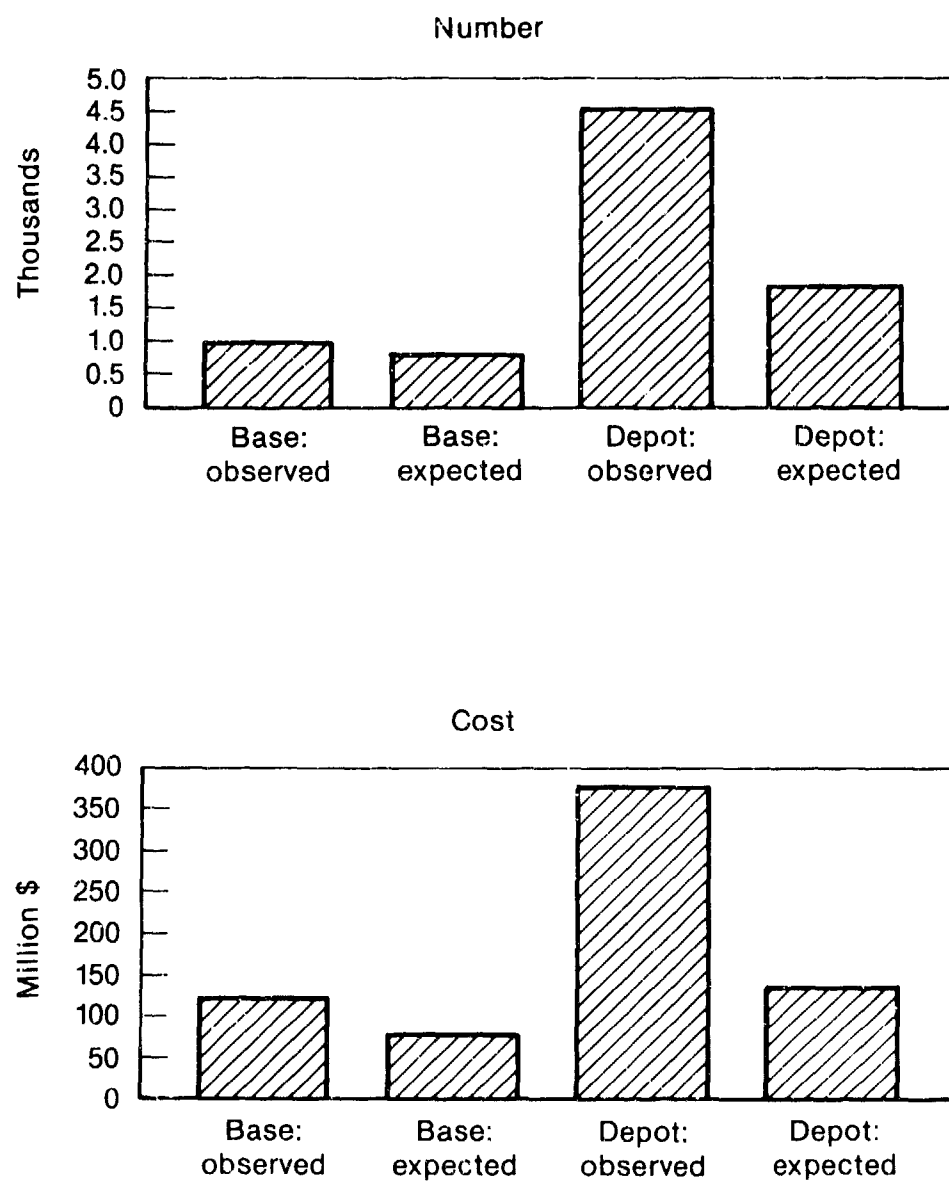


Fig 19—F-15: Number in and cost of LRU pipeline,  
critical 84 peculiar parts

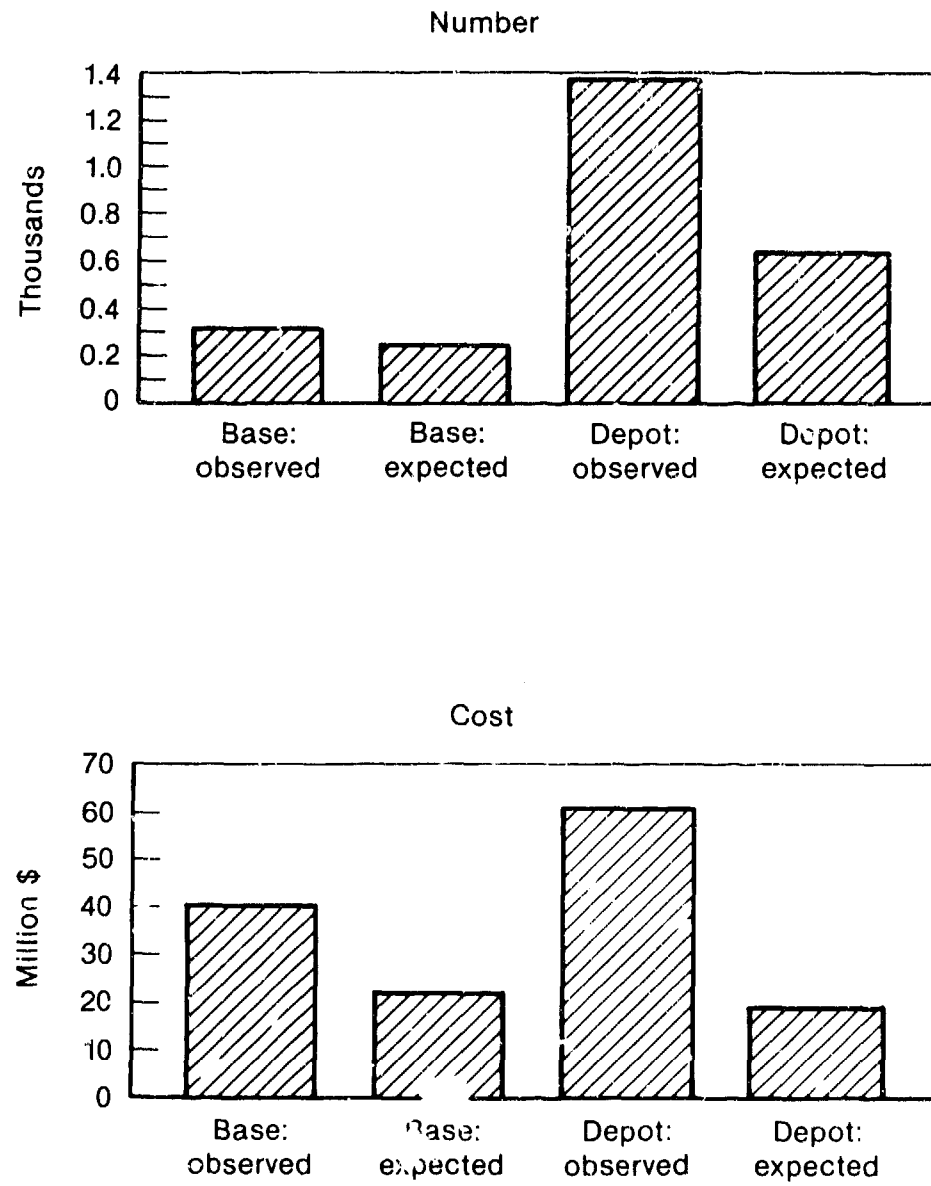


Fig. 20-F-15: Number in and cost of LRU pipeline,  
critical 35 peculiar parts



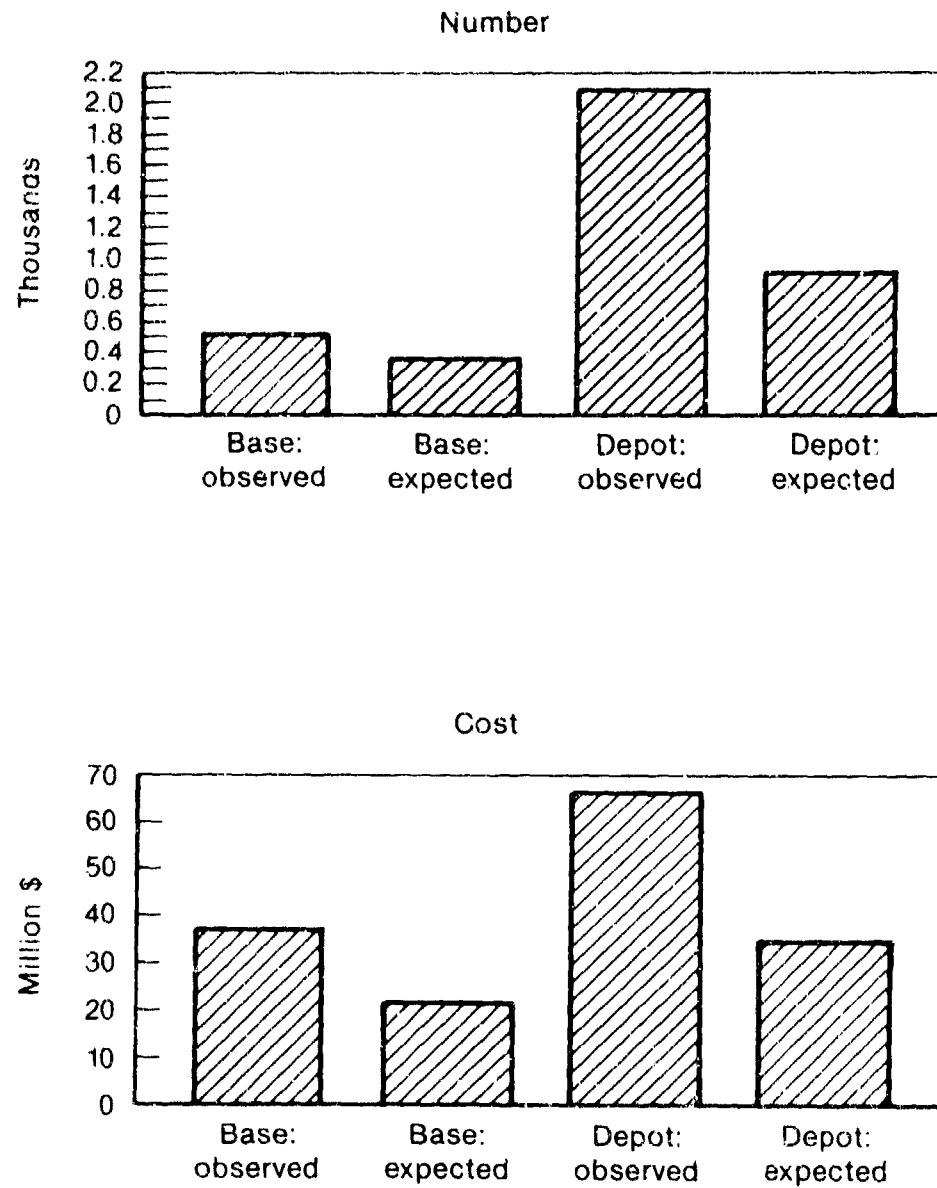


Fig. 21-F-15: Number in and cost of LRU pipeline,  
MICAP (23) peculiar parts

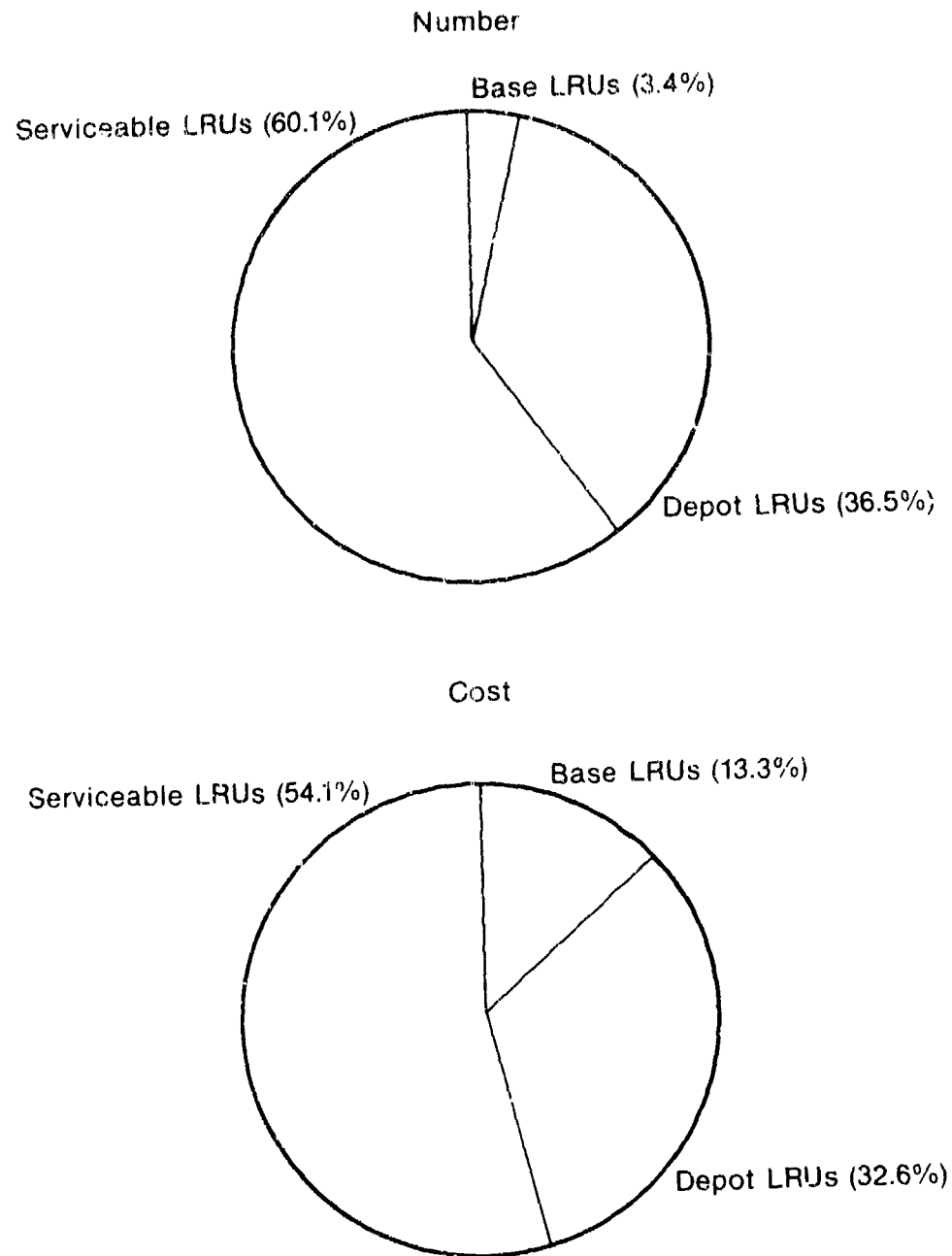


Fig. 22—Serviceability of components, F-15 all (1039)

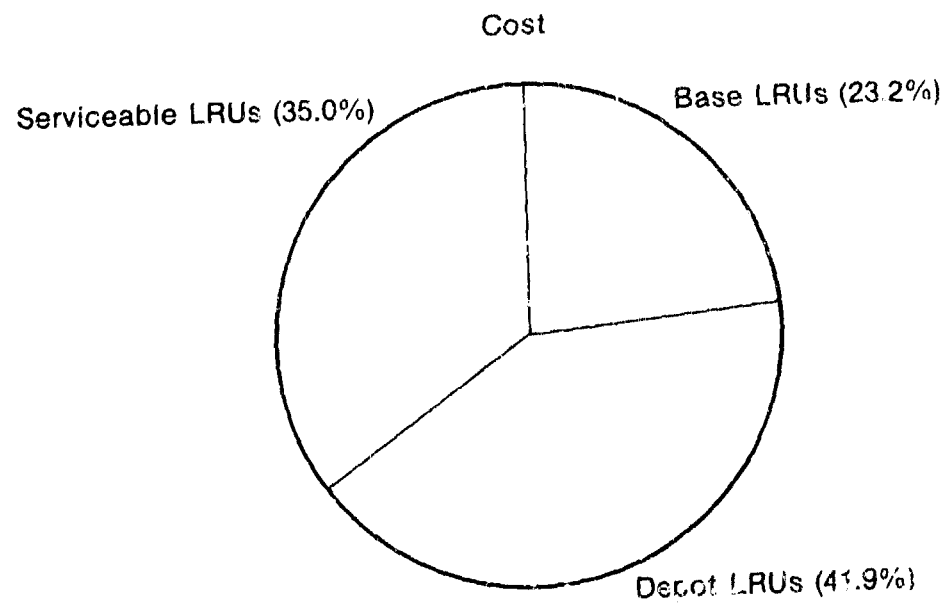
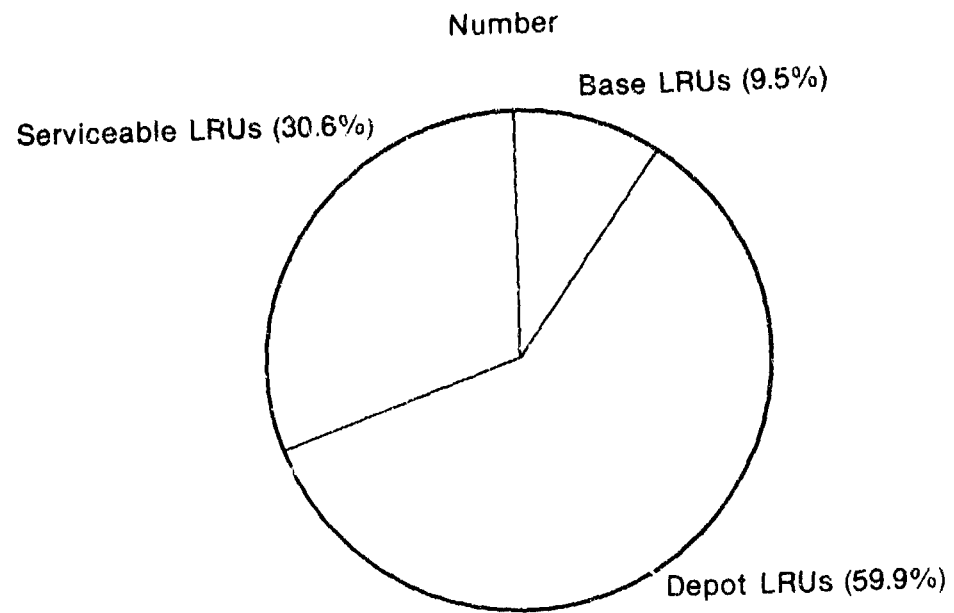


Fig. 23—Serviceability of components, F-15 BAD 144

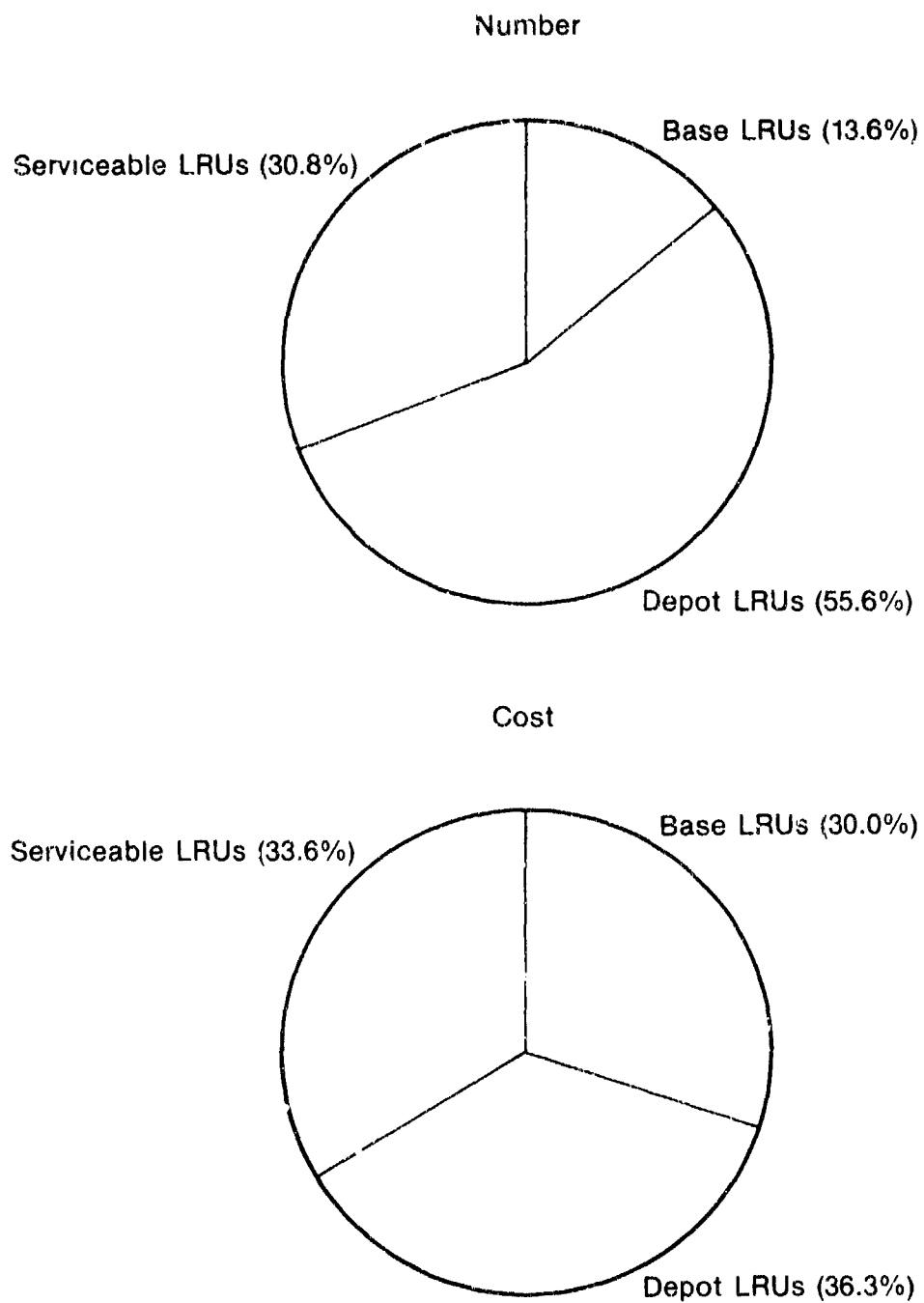


Fig. 24—Serviceability of components, F-15 BAD 57

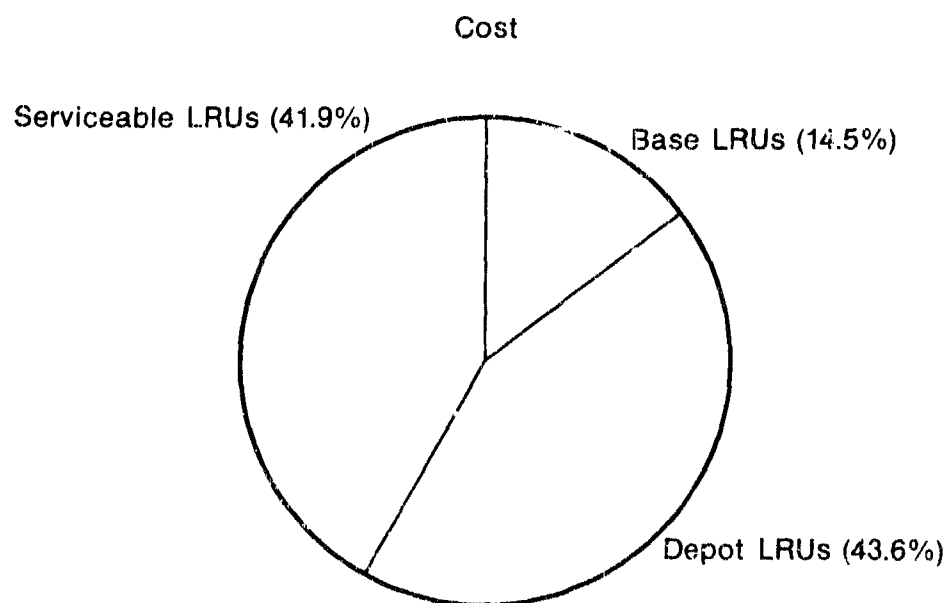
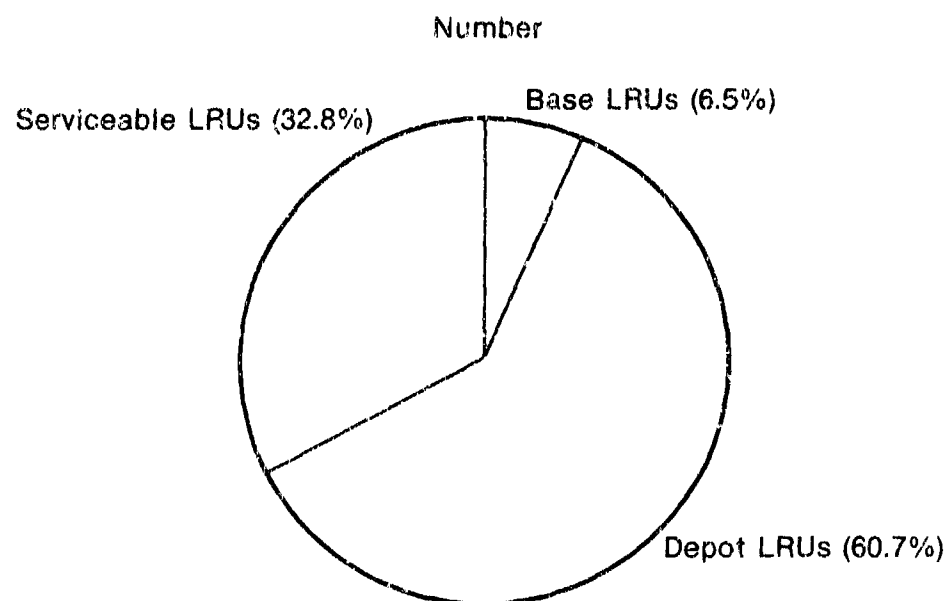


Fig. 25—Serviceability of components, F-15 MICAP (46)

Why are there so many unserviceable assets at the depot? In part this may result from "breakdowns" in the repair process: shortages of EOQ items, SRUs, or repair equipment failures. In part it may also result from the accumulation of obsolete SRUs and LRUs. But in part this appears to be a direct result of the depot quarterly repair computation, which is based on a presumed ability to predict needs. Although that computation is complicated and done on a cumulative basis, on a quarterly basis it reduces to essentially the following formula:

$$\begin{aligned} \text{Quarterly Repair Requirements} &= \text{Expected Demands} \\ &- \text{Expected Base Repairs} - \text{Serviceables.} \end{aligned}$$

This computation has an interesting ramification when one considers the earlier evidence that demand rates are constantly changing and the factors used in this computation are in a continual state of flux: Suppose, for instance, there is a part that has been fairly stable over several years with a constant demand rate. Then the expected demands will stay the same; and if the condemnation rate is very low, the number of serviceables will remain the same, as will repairs at the base. We may expect that the system has approximately the correct number of assets. Now, suppose the demand rate for this part increases over a period of several quarters or a year. Expected demands will go up, number of serviceables will go down, and the expected quarterly repair requirement will show that more carcasses should be repaired at the depot. If the additional carcasses are not available, a common situation in a steady-state environment such as has been described for this hypothetical part, this lack of assets will result in a procurement. Now suppose that during or after the procurement lead time, the demand rate for the asset drops back to its previous lower level, and stays there. Consider what has happened now: Expected demands are back where they were. The number of serviceables has shot up in accordance with the buy, and the number of repairs at the base remains the same as it was. Hence the quarterly repair requirement at the depot will be diminished by approximately the number of assets that were purchased. In other words, the way the depot repair requirement is calculated assures that if the demand rate for this part drops back to its pre-buy level, those serviceable assets that were procured will percolate around the system until they become unserviceable at the depot. If the demand rate stays at its pre-buy level, that is where they will remain.

In the philosophies currently embodied in the AFLC requirements and capability assessment modeling, it is correctly realized that every serviceable asset can possibly result in an increase in aircraft availability, although probabilistically that increase may be infinitesimally

small. The procurement philosophy is to rank parts by the ratio of their probable increase of aircraft availability to their cost. The parts having the highest ratio are those whose procurement is likely to result in the biggest improvement in aircraft availability for each dollar spent.

This same philosophy can, and should be, applied to the selection of parts for depot repair.<sup>5</sup> The per dollar increases to availability that can be achieved from more repair substantially exceed the levels of availability per dollar that are being funded in procurement programs.<sup>6</sup>

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<sup>5</sup>This idea of ranking parts to be repaired by their contribution to aircraft availability is behind the DRIVE (Distribution and Repair In Variable Environments) model developed at RAND that is currently being implemented with the help of the staff at the Ogden Air Logistics Center.

<sup>6</sup>Personal communication from AF/LEXY.

## V. CONCLUSIONS

Although both depot and base maintenance attempt to mitigate the peaks in the demand process (and for that matter, long repair pipelines whatever their cause) by taking such actions as repairing faster, it is difficult to see the success of these efforts in these data. In fact, these data indicate that pipeline contents are more variable than the demand processes. To some extent this is the result of breakdowns in the repair process. In other words, when possible, maintenance may work faster to mitigate peaks in the repair pipeline caused by peaks in the demand process, but breakdowns (including EOQ shortages and repair equipment failures) in the maintenance process may also cause many of the peaks in the repair pipelines. To the extent that that is true, maintenance induced peaks in pipeline contents also have a deleterious effect on aircraft availability.

In addition to the direct effects of pipeline size and variability on aircraft availability, arrival processes and pipeline contents having large VTMRs and shifting means wreak havoc with the ability to compute requirements and assess capability. On the one hand, the bunching of demands in the repair pipeline after a failure in the repair process violates the assumption of independence between the repair process and the arrival process. On the other hand, an arrival process with a large VTMR or a seemingly changing mean is probably the result of several related phenomena, all unmodeled in our current requirements models:

1. Removals occur in clusters.
2. Sorties are not independent.
3. Disjoint time intervals are not independent.
4. We just simply do not know the clock. Something other than sorties and flying hours drives removals.

All of these explanations are probably important. Unfortunately, none of them is mirrored in the standard approach to capability assessment modeling or computing requirements.

Regarding the depot pipeline contents: Policies must be reoriented. Current policy results in prohibitively expensive repair pipelines and reduced aircraft availability. Several explanations have been suggested for these long pipelines. Depot policies, decisions, and goals should be aimed at reducing these pipelines and increasing aircraft availability.



## Appendix A

### THE 19 PARTS DATA

The following tables give the demands per 1000 flying hours for the 19 parts, at Luke and Holloman AFBs, by calendar year and quarter. Notes recount the comments of base maintenance personnel at Luke and Holloman regarding the part and its characteristics.

The comments may help explain the variation, or lack of it, in the demand stream. The comments are subjective impressions, and their effects may or may not be evident in the data. Even though a perception may be accurate, the effect may be swamped by other types of failures, hence not apparent in the data.

Following the tables is a summary chart giving the raw data and some preliminary calculations of VTMRs.

Table A.1

74FQO, RADAR DATA PROCESSOR; DIGITAL PROGRAMMABLE SIGNAL  
PROCESSOR, BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	9.3	16.1	16.9	13.7	6.8	16.7	11.3	10.9	14.5	21.0
Holloman AFB			11.3	17.6	10.4	24.2	29.6	25.5	48.7	13.8

R = 7.3

LUKE: There was a modification on this part late in CY 1981 that resulted in an increased demand rate. Additionally, there is a seasonal effect. The mission type may also influence demand.

HOLLOMAN: The seasonal effect was also mentioned. In May 1982, the internal software was revised, resulting in the removal of the LRU from every aircraft in the wing to load the new program. This action inadvertently resulted in a recorded demand for every LRU affected.

Table A.2

75MCO, CONVERTER PROGRAMMER (DIGITAL COMPUTER),  
BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	6.2	4.7	5.3	7.9	9.2	8.6	3.4	5.2	6.4	5.7
Holloman AFB		1.7	1.5	0.4	2.7	4.7	3.4	13.3	7.6	10.9

The Converter Programmer provides the interface between the weapon control system and air-to-air missiles.

Demand rates at Bitburg have been reported to be eight times higher than in TAC because of the USAFE practice of periodically uploading the radar missile (AIM-7) and running a full test of the Converter Programmer.

LUKE: It was said that mission type may affect variability, but the staggered training program at Luke keeps the mission program fairly constant over three-month time periods.

HOLLOMAN: COSO was implemented at Holloman in 1 Feb 82. The Converter Programmer has a high CND rate; it is likely that before COSO most suspected failures, and many true failures, were tested, repaired, and replaced without proper documentation, thus understating the demand on supply.

Table A.3

13AJB, MAIN LANDING GEAR WHEEL, BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	70	90	68	60	56	59	42	53	48	49
Holloman AFB							50	98	25	50

R = 5.5

The Main Landing Gear Wheel has been, and continues to be, a problem for the F-15.

LUKE: The clustering of demands for this problem part may be the result of taking a hard look at all wheels whenever new problems are discovered.

HOLLOMAN: The Air Force practice of scheduling aircraft so that they age at comparable rates may cause near simultaneous wearout of such components as wheels. (The practice may result in the perception among maintenance personnel that parts are simultaneously wearing out, resulting in the bunching of demands.)

Table A.4

14DDA, ROTARY HYDRAULIC ACTUATOR, RUDDER, RACK AND PINION  
ACTUATOR (TWO PER AIRCRAFT), BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	1.2	2.2	1.8	2.6	2.8	1.4	3.8	3.5	2.5	3.7
Holloman AFB			.8	.6	.9	1.6	1.3	.5	9.7	3.1
R = 1.7										

LUKE: The actuator was modified to give increased throw. During the transitions to new actuators, an actuator failure typically resulted in two actuator replacements.

HOLLOMAN: The teeth on the rack were a problem, but seasonal hydraulic leakage did not seem to be much of a problem. The chipped teeth are typically discovered during phase inspection. The modified actuator is stronger. The F-15 C and D models have a different actuator. (Luke has only A and B models.)

Table A.5

14CDA, STABILATOR, HYDRAULIC SERVO CYLINDER,  
BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	2.7	2.9	7.1	4.0	4.6	2.5	6.9	6.2	4.7	4.9
Holloman AFB	1.4	.9	2.3	2.9	4.7	3.3	5.6	5.0	5.2	4.2
R = 3.0										

LUKE: There was a leakage problem. In response, the O-ring seal was improved and bases were given increased repair authority. (In the case of a slowly deteriorating seal, the increased repair authority may result in an increase in demands.)

HOLLOMAN: A change in the technical data, which extended the acceptable limitations on leaks.

Table A.6

24ACO, JET FUEL STARTER (AUXILIARY TURBINE ENGINE),  
BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	3.3	4.3	3.6	2.3	4.2	4.7	3.3	2.0	2.8	3.6
Holloman AFB		3.1	1.5	1.6	.9	2.4	1.3	3.4	2.3	4.6

R = 1.2

LUKE: Hot weather makes engine starting more difficult, increasing the load on the starter; also, cooling is more difficult, and the required tolerances are tighter. Problems are often induced by student pilots.

HOLLOMAN: At best the starter unit is "marginal." If repair parts are available, some faulty units may be repaired on flight line, thus the lack of availability of repair parts may drive up the recorded demands for the LRU.

Table A.7

74FSO, RADAR TARGET DATA ANALOGUE PROCESSOR,  
BY CALENDAR YEAR QUARTER  
(Works in Conjunction with HUD)

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	8.3	13.2	13.8	9.2	6.8	10.3	8.4	8.8	15.7	17.5
Holloman AFB		11.0	15.6	9.0	9.6	14.3	11.5	36.6	8.2	12.3

R = 6.6

LUKE: High temperatures result in increased demands in the third quarter. In addition, the test station is erratic, and demands may rise and fall with the perceived ability of the test station to isolate ambiguous faults. Demands were probably underrecorded before COSO.

HOLLOMAN: In the first quarter of 1982, a modification made the unit incompatible with certain other unmodified LRUs, resulting in a demand for a modified processor whenever the other LRUs were replaced with modified versions.

Table A.8

74FUO, RADAR ANTENNA, MOVING DISH, HYDRAULICALLY POSITIONED,  
BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	10.6	10.8	14.1	10.9	17.4	18.4	11.3	9.3	11.2	17.2
Holloman AFB	9.6	21.7	10.8	13.5	11.1	21.4	15.8	10.2	9.2	12.1
R = 4.8										

LUKE: Demands should track with demands for the power supply. There may be a seasonal effect resulting from seal leakage during periods of extreme temperatures. A leaking seal is not wartime critical and may be repaired locally. The implementation of COSO probably increased the recording of demands.

HOLLOMAN: The software in the driving computer sometimes resulted in excessive antenna swings. The problem was fixed in the 3rd quarter of 1981, resulting in fewer demands. There are seasonal problems with seals.

Table A.9

74JAO, AIR NAVIGATION MULTIPLE INDICATOR, COCKPIT CRT DISPLAY,  
BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	6.6	5.6	6.3	7.3	6.8	4.9	4.7	4.6	5.7	6.8
Holloman AFB							8.2	5.7	5.8	4.4
R = .9										

LUKE: CND failures are rare, hence fault isolation is easier and the result may be more regularity in the demand stream.

HOLLOMAN: Many pulls are for faded (sun bleached) scope screens. (F-15s are left on the ramp without canopy covers or scope covers.) This type of failure may induce a certain amount of regularity to the demand stream.

Table A.10

74KEA, 74KEB, 74KEC, HUD CAMERA BODY, AND TWO ELECTRIC MODULES,  
BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Body	(R = .6)									
Luke	1.9	0.9	0.7	1.9	0.9	1.5	1.2	1.3	1.5	1.7
Holloman			.4	1.0	.6	.6	2.4	1.3	4.2	1.0
Module B	(R = 1.6)									
Luke	2.1	2.7	2.9	2.0	1.7	3.0	3.5	2.3	1.9	4.2
Holloman	(NSN not loaded)									
Module C	(R = 1.8)									
Luke	1.9	2.0	3.6	3.0	2.3	1.0	1.7	2.8	1.9	3.9
Holloman						2.7	3.1	1.3	5.8	3.1

HOLLOMAN: Demands are usually recorded when aircraft are being prepared for deployments. The body and modules are SRUs. Unit is used to score missions at Red Flag and WESEP and comparable exercises, but is not war-mission critical. In the second quarter of 1982, 16 aircraft were deployed to Red Flag and seven to WESEP. Red Flag sometimes requires 32 aircraft.

Table A.11

71AEO, INERTIAL MEASUREMENT UNIT, BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-3	82-4
Luke AFB	4.8	3.6	9.4	9.8	12.8	10.0	9.1	12.2	5.4	9.4
Holloman AFB							16.3	12.8	24.0	15.1

R = 5.4

HOLLOMAN: Demands peak in cold weather because of lack of warm up. Pilot experience and technique contribute to variability.

Table A.12

## 74FHO, RADAR POWER SUPPLY, BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	6.8	9.9	10.0	10.1	9.1	9.3	9.5	6.5	14.8	18.2
Holloman AFB	7.6	10.5	6.7	8.2	11.1	14.7	17.3	15.7	14.0	17.3

R = 6.9

LUKE: Demands are influenced by changes in technical data and software. Flight line inexperience in fault isolation may also drive up demands.

HOLLOMAN: The LRU is easy to pull and gets pulled often (especially in conjunction with the antenna—fault isolation between these two LRUs is difficult). It has a high UND rate.

Table A.13

## 41ACV, BLEED AIR SHUTOFF VALVE, BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	2.5	3.4	3.4	0.9	5.8	3.7	2.1	3.0	4.5	7.1
Holloman AFB			2.9	2.0	1.7	1.4	2.9	4.5	13.8	6.0

R = 5.4

Controls bleed air to environmental systems. There is one valve on each engine. The valves are inaccessible with the engine in place; it must be slid aft for access.

LUKE: This is a critical item and is in chronic short supply. Registered demands may follow availability. (As of the fall of 1985, serviceable valves, although not expensive, were still in short supply.)

HOLLOMAN: There was a TCTO to replace spring. Replacements occurred during engine pulls or inspection. The new spring has not enhanced the valve's reliability.

Table A.14

## 46ADE, HEAT EXCHANGER, BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	4.2	3.1	3.3	3.7	4.9	7.4	1.8	1.5	6.8	11.1
Holloman AFB		2.6	2.1	.6	1.7	3.3	3.5	5.9	3.3	6.5

R = 10.2

Aircraft has two for cooling lubricating oil and hydraulic fluid. The heat exchanger uses fuel from the center tank as a heat sink and pumps the warmed fuel into the wing tanks for cooling. The thermal sensors in the exchangers work independently. If one exchanger activates more frequently than the other, a wing heavy condition will result, restricting the aircraft's flight envelope. (A modification to the aircraft that resulted in the need for a more balanced configuration also resulted in an increase in the demand rate for heat exchangers.)

LUKE: A failure of the accessory drive often causes contamination and heat exchanger failure. Summer increases use, hence the likelihood of failure, and the severity of the out-of-balance problem.

HOLLLOMAN: In 1982 a new thermal (sensing) element was introduced. The problem did not improve, it actually got worse.

Table A.15

## 24ANO, CENTRAL GEAR BOX, BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	4.5	4.5	5.1	3.6	4.6	5.3	2.0	2.5	2.5	4.9
Holloman AFB		2.1	1.9	2.2	1.9	2.8	2.9	6.4	2.7	4.8

R = 1.9

Takes power output from main engines and/or Jet Fuel Starter and drives generators and hydraulic pumps.

LUKE: The demand pattern should track with the Jet Fuel Starter.

HOLLLOMAN: A common cause of failure is a pawl shaft that loses teeth. The LRU may be repaired on the flight line if the pawl shaft is available; if not, pawl shaft failure will result in a demand for the LRU.



Table A.16

## 23HAA, UNIFIED TURBINE CONTROL, BY CALENDAR YEAR QUARTER

Base	80-2	80-3	80-4	81-1	81-2	81-3	81-4	82-1	82-2	82-3
Luke AFB	5.8	6.5	4.0	5.3	5.2	7.3	3.9	9.8	7.4	5.9
Holloman AFB		6.2	12.1	10.2	5.1	6.7	7.3	8.5	4.8	8.7

R = 2.4

Fuel flow control for engines and Jet Fuel Starter. Large, heavy, cumbersome.

LUKE: Demands seem high every 3rd quarter. Combination of equal aging of aircraft and depot maintenance "batching" overhauls may result in periodicity. (Recall that Luke gets lower priority than many of the bases and may be more likely to get "batch" shipments of parts that are often in short supply.)

HOLLOMAN: Unit has a "stepper" electric motor that is prone to developing shorts. Depot has developed a fix that results in increased MTBF. A foam fire retardant used in fuel tanks may deteriorate with age and result in clogged filters. A fuel control with clogged filters must be returned to the depot for maintenance.

Table A.17  
SUMMARY OF THE 19 PARTS DATA

Part No.	Quarter														Total	Hrds	N	Mean	Rate	V.M.R.	X <sup>2</sup>
	2/80	3/80	4/80	1/81	2/81	3/81	4/81	1/82	2/82	3/82	4/82	1/83	2/83								
1280010423952	-9	7	7	2	14	24	19	14	54	57	51	83	58	450	62941	12	37.5	7.1	18.9	196.0	0.0000
1630010054262	-9	15	124	74	37	154	275	546	152	260	213	180	178	2208	62941	12	184.0	35.1	95.1	1748.4	0.0000
1650000000128	-9	4	3	5	8	7	3	58	16	45	25	25	0	199	62941	12	16.6	3.2	19.3	212.8	0.0000
1650010414568	-9	6	11	10	7	8	11	8	-9	-9	-9	-9	-9	61	35551	7	8.7	1.7	9.4	2.3	0.6978
1650010790503	-9	7	4	11	15	25	31	28	31	68	17	19	26	282	62941	12	23.5	4.5	11.1	122.2	0.0009
1660010155017	-9	14	10	9	7	16	16	25	83	31	16	28	38	293	62941	12	24.4	4.7	14.1	155.0	0.0005
2835010344772	-9	13	7	8	5	12	7	19	14	24	22	15	28	185	62941	12	15.4	2.9	4.9	53.4	0.0000
2835010346948	-9	9	9	11	10	14	16	36	16	25	21	19	32	218	62941	12	18.2	3.5	4.0	44.1	0.0000
2915010167217	-9	26	58	52	27	34	40	50	29	45	-9	-9	-9	361	46819	9	40.1	7.7	3.8	30.6	0.0002
2915010653500	-9	11	10	3	9	17	19	33	20	31	14	12	26	205	62941	12	17.1	3.3	4.5	43.8	0.0006
5841010447134	-9	54	90	55	123	155	163	143	94	52	36	47	58	1074	52941	12	89.5	17.1	22.4	246.8	0.0000
5841010505979	-9	38	44	32	42	59	95	88	84	90	53	56	59	740	62941	12	61.7	11.8	6.0	66.2	0.0000
5841010568180	-9	46	75	46	51	73	63	205	49	12	10	21	10	661	62941	12	55.3	10.5	47.6	524.0	0.0000
5841010630855	-9	48	91	52	69	59	87	57	55	63	71	66	71	789	62941	12	65.8	12.5	3.4	36.9	0.0001
6605010370410	-9	28	36	17	27	37	45	40	35	47	45	38	0	395	62941	12	32.9	6.3	5.6	61.1	0.0000
6605010954208	-9	37	73	51	31	63	45	135	77	73	102	99	24	810	62941	12	67.5	12.9	14.3	157.0	0.0000
6620001487306	-9	0	0	0	20	17	19	29	11	23	17	12	12	160	62941	12	13.3	2.5	6.3	69.4	0.0000
6770010030318	-9	4	6	10	4	14	14	9	39	17	22	14	21	174	62941	12	14.5	2.8	5.5	60.7	0.0000
6710010200400	-9	0	0	0	14	17	17	7	35	16	18	11	19	154	62941	12	12.8	2.4	7.0	16.9	0.0000
6710010200408	-9	2	5	3	3	13	13	7	25	5	17	13	13	112	62941	12	9.9	1.9	4.2	45.7	0.0000
TOTAL	-9	369	663	451	523	822	598	1727	929	993	821	821	722	9709	62941	12	809.1	154.1	101.7	1119.2	0.0000
FLYING HOURS	-9	6172	4791	5145	5242	5747	5533	5526	6035	5233	4692	6220	5140								

NOTE: -9 means missing data

NOTE: -9 means missing data

Table A.17—continued

Port No.	Quarter																Total	Hours	N	Mean	Rate	VIMK	X <sup>2</sup>	p	
	2/80	3/80	4/80	1/81	2/81	3/81	4/81	1/82	2/82	3/82	4/82	1/83	2/83												
1280010423952	9	5	5	28	19	23	30	-9	54	78	51	103	30	435	49957	12	36.3	8.7	13.0	143.2	6.0009				
1630010024262	15	32	56	75	196	367	347	-9	254	265	144	149	252	2212	49957	12	104.3	44.3	57.4	631.5	0.0006				
1650000000128	4	6	5	3	7	7	5	-9	15	17	19	23	9	120	49957	12	10.0	2.4	1.0	20.1	0.0442				
1650010454568	1	10	3	5	11	9	3	-9	-9	-9	-9	-9	-9	12	21798	7	6.6	1.9	3.1	10.9	0.0044				
1650010730503	11	19	4	10	10	19	25	-9	10	23	47	24	27	229	49957	12	19.1	4.6	3.1	34.5	0.0003				
1660010155017	6	7	4	9	9	10	13	-9	16	14	12	26	47	173	49957	12	14.4	3.5	3.9	43.1	0.0006				
2835010544714	4	5	2	8	2	8	8	-9	18	16	14	10	18	113	49957	12	9.4	2.3	1.4	15.5	0.1621				
2835010345948	3	5	2	20	11	6	10	-9	17	20	19	18	28	155	49957	12	13.3	3.2	3.7	45.0	0.0000				
2915010137217	17	31	23	24	12	32	29	-9	14	35	33	20	-9	273	43692	11	24.9	4.3	4.4	44.2	0.0000				
2915013653500	7	3	1	10	6	18	12	-9	-9	20	25	16	10	136	44355	11	12.4	3.1	3.8	38.3	0.0000				
5841010447134	39	103	32	51	61	113	85	-9	71	63	69	59	54	800	49957	12	66.7	16.0	19.8	218.2	0.0000				
5841010505279	17	30	27	47	27	59	67	-9	97	26	71	73	57	658	49957	12	54.8	13.2	4.7	51.2	0.0007				
5841010508180	20	34	23	52	41	57	46	-9	78	13	29	35	11	436	49957	12	36.3	6.7	16.3	175.2	0.0070				
58410106310875	0	2	34	57	52	65	49	-9	6	112	71	73	59	583	49957	12	48.6	11.7	18.6	245.1	0.0000				
6605010370410	7	21	12	31	22	30	24	-9	14	41	37	31	33	323	49957	12	26.9	6.5	3.2	34.7	0.0003				
6605010954208	37	62	14	62	57	107	80	-9	147	134	153	201	126	1180	49957	12	98.3	23.6	11.3	124.7	0.0000				
6620001487306	-9	-9	-9	13	12	9	10	-9	20	20	16	14	19	133	41661	9	14.0	3.2	1.0	8.0	0.4288				
6710010303318	5	4	2	5	9	16	16	-9	13	7	18	5	12	116	49957	12	9.7	2.3	2.9	21.9	0.0250				
6710010700400	1	5	6	5	7	12	10	-9	-9	13	11	15	13	98	44355	11	8.9	2.2	0.7	6.8	0.7465				
6710010213408	0	0	3	3	3	7	4	-9	-9	4	3	2	7	43	44355	11	3.9	1.0	1.2	12.2	0.1100				
TOTAL	211	443	268	518	575	974	873	-9	212	548	854	911	863	8416	49957	12	701.7	168.5	53.8	567.5	0.0000				
FLYING HOURS	2576	2447	1473	2100	3165	3661	4316	-9	3400	5417	3297	5518	6265												

NOTE: -9 means missing data

NOTE: -9 means missing data

Table A.17—continued

Part No.	Quarter												Total	Hours	N	Mean	Rate	VTRR	$\chi^2$	P	
	2/80	3/80	4/80	1/81	2/81	3/81	4/81	1/82	2/82	3/82	4/82	1/83									2/83
1280010423952	-9	5	8	1	9	4	8	2	3	3	3	6	7	59	9398	12	4.9	6.3	1.5	16.5	0.1238
1630010054262	-9	25	27	31	12	2	38	46	11	41	7	28	29	297	9398	12	24.8	31.6	8.2	90.7	0.0000
1650000000128	-9	1	0	7	4	0	5	1	5	1	2	5	8	39	9398	12	3.3	4.1	2.1	23.5	0.0152
1650010414566	-9	1	2	3	3	2	2	0	-9	-9	-9	-9	-9	13	5264	7	1.9	2.5	0.6	3.5	0.7485
1650010790503	-9	1	0	2	0	0	0	0	-9	2	6	8	12	31	8595	11	2.8	3.6	5.1	50.7	0.0000
1660010155017	-9	1	1	3	1	0	8	2	4	8	7	9	12	56	9398	12	4.7	6.0	2.9	31.9	0.0008
2835010344772	-9	1	4	1	1	2	1	2	12	1	2	5	2	34	9398	12	2.8	3.6	3.4	37.2	0.0001
2835010346948	-9	3	2	1	2	5	3	2	4	7	1	6	4	40	9398	12	3.3	4.3	1.0	11.0	0.4450
2915010167217	-9	5	9	1	3	5	6	0	2	2	-9	-9	-9	33	6860	9	3.7	4.8	2.4	19.3	0.0133
2915010653500	-9	1	3	6	0	3	2	3	5	6	6	3	5	43	9398	12	3.6	4.6	1.0	11.4	0.4120
5841010487134	-9	2	4	3	9	5	18	12	20	7	3	7	7	97	9398	12	8.1	10.3	4.1	45.1	0.0000
5841010505979	-9	9	8	8	10	11	13	10	7	1	4	5	5	91	9398	12	7.6	9.7	1.7	18.8	0.0644
5841010588180	-9	13	9	6	4	19	24	4	4	4	1	1	0	89	9398	12	7.4	9.5	8.1	99.2	0.0000
5841010630855	-9	20	6	6	9	4	33	9	6	6	11	13	13	116	5398	12	11.3	14.5	6.3	69.8	0.0000
6505010370410	-9	4	0	0	7	5	3	7	3	3	2	4	4	42	9398	12	3.5	4.5	1.4	15.9	0.1443
6605010954208	-9	14	10	3	7	11	20	8	18	6	8	16	11	132	9398	12	11.0	14.0	2.4	26.9	0.0048
6620001487306	-9	0	1	3	1	3	6	1	0	3	2	1	2	23	9398	12	1.9	2.4	1.4	15.9	0.1447
6710010030318	-9	0	1	5	1	4	24	6	4	3	0	2	9	59	9398	12	4.9	6.3	8.7	96.0	0.0000
6710010200400	-9	2	1	4	1	0	11	2	3	4	2	5	7	42	9398	12	3.5	4.5	2.5	27.9	0.0033
6710010200408	-9	0	0	8	8	2	2	1	0	7	1	3	2	34	9398	12	2.8	3.6	3.4	36.2	0.0001
TOTAL	-9	108	96	102	92	87	227	118	116	117	74	133	145	1415	9398	12	117.9	150.6	13.1	144.5	0.0000
FLYING HOURS	-9	653	715	766	766	821	774	762	803	793	810	841	887								

NOTE: -9 means missing data.

NOTE -9 means missing data

Table A.17--continued

Port No.	Quarter												Total	Hours	N	Mean	Rate	V/MR	$\chi^2$	P	
	2/80	3/80	4/80	1/81	2/81	3/81	4/81	1/82	2/82	3/82	4/82	1/83									2/83
1280010423952	42	21	25	6	39	24	44	51	52	78	61	60	45	548	71/89	13	42.2	7.6	5.2	62.9	0.0000
1630010054262	54	44	16	31	50	32	9	15	27	8	14	7	2	309	71/89	13	23.8	4.3	22.5	269.5	0.0000
16500010000128	6	5	3	18	11	10	18	15	8	15	19	29	25	182	71/89	13	14.0	2.5	2.7	32.2	0.0013
1650010414568	21	6	8	6	8	16	13	-9	-9	-9	-9	-9	-9	78	31621	7	11.1	2.5	2.8	17.0	0.0091
1650010790503	14	12	19	17	15	9	16	20	12	23	22	15	58	252	71/89	13	19.4	3.5	5.3	63.1	0.0000
1660010155017	7	10	10	6	12	20	13	11	18	18	11	14	18	168	71/89	13	12.9	2.3	0.9	10.9	0.5369
2835010344772	8	13	6	16	7	17	11	8	12	10	8	7	8	131	71/89	13	10.1	1.8	2.3	27.4	0.0069
2835010346948	11	14	20	9	17	16	20	8	15	10	17	9	17	183	71/89	13	14.1	2.5	2.5	30.2	0.0027
2915010167217	23	13	10	18	13	12	15	5	10	11	-9	-9	-9	130	51263	10	13.0	2.5	3.8	33.9	0.0097
2915010653500	5	5	6	3	12	17	18	12	7	15	7	1	3	111	71/89	13	8.5	1.5	3.9	47.3	0.0000
5841010447134	61	72	45	36	13	48	57	44	37	26	21	18	19	497	71/89	13	38.2	6.9	21.1	252.7	0.0000
5841010505979	40	42	42	22	25	33	31	49	27	34	41	24	19	429	71/89	13	33.0	6.0	7.3	87.3	9.0090
5841010588180	43	47	28	13	25	37	5	11	23	37	43	42	34	388	71/89	13	29.8	5.4	18.4	124.4	0.0000
5841010630855	57	42	58	53	46	56	30	34	55	84	59	59	74	707	71/89	13	54.4	9.6	4.3	51.6	0.0000
6605010370410	4	8	12	14	9	27	23	27	12	23	21	23	21	274	71/89	13	17.2	3.1	2.4	28.6	0.0045
6605010954208	45	43	44	12	23	46	37	74	76	59	111	104	135	869	71/89	13	62.2	11.3	11.2	134.6	0.0000
6620001447306	2	9	5	30	22	23	10	10	4	12	13	17	15	172	71/89	13	13.2	2.4	5.8	69.4	0.0030
6710010030318	7	3	7	20	10	12	2	6	11	15	7	7	5	112	71/89	13	8.6	1.6	3.1	37.8	0.0002
6710010200400	9	9	9	16	8	13	8	0	10	10	20	5	3	120	71/89	13	9.2	1.7	3.3	40.2	0.0001
6710010200408	4	6	9	10	12	12	7	9	5	9	3	6	3	95	71/89	13	7.3	1.3	2.2	25.9	0.0111
TOTAL	463	424	382	356	377	480	387	423	440	513	532	486	539	5796	71/89	13	445.8	83.7	20.2	241.8	0.0000
FLYING HOURS	4103	3161	3999	4195	4824	5507	5212	5674	7225	6413	6813	6629	7084								

NOTE: -9 means missing data

Table A.17—continued

Port No	Carrier													Total	Hours	N	Mean	Rate	VTRM	$\chi^2$	P
	2/80	3/80	4/80	1/81	2/81	3/81	4/81	1/82	2/82	3/82	4/82	1/83	2/83								
1280010423952	30	21	29	51	60	51	23	36	44	39	15	22	41	462	82466	13	35.5	5.6	5.1	60.9	0.0000
16300104054262	335	399	317	383	364	349	277	367	329	319	51	233	237	4020	82466	13	309.2	48.7	50.7	608.4	0.0000
1650000000120	6	10	10	17	18	8	25	24	17	7	9	13	15	179	82466	13	13.8	2.2	2.1	25.4	0.0129
1650010414568	6	8	9	13	1	6	6	-9	-9	-9	-9	-9	-9	49	40476	7	7.0	1.2	2.0	12.1	0.0590
1650010190503	13	13	39	26	30	15	46	43	52	7	7	13	41	325	82466	13	25.0	3.9	7.1	85.1	0.0000
1660010155017	12	15	19	6	38	22	14	21	31	48	29	45	58	358	82466	13	21.5	4.3	6.3	76.1	0.0000
2835010344772	16	19	20	15	27	28	22	14	19	23	27	25	32	287	82466	13	22.1	3.5	1.1	13.1	0.3587
2835010346948	23	20	28	23	30	34	20	17	17	32	25	23	27	319	82466	13	24.5	3.9	1.8	21.2	0.0475
2915010167217	28	29	22	37	34	43	26	19	51	38	-9	-9	-9	327	60696	10	32.7	5.4	2.7	24.3	0.0038
2915010653500	20	14	18	24	32	44	12	16	47	72	9	9	9	320	82466	13	24.6	3.9	15.3	183.0	0.0000
5841010447134	45	72	93	88	44	99	75	75	99	49	25	19	45	828	82466	13	63.7	10.0	15.1	180.6	0.0000
5841010505979	33	44	55	65	59	55	63	45	101	110	52	76	70	828	82466	13	63.7	10.0	5.6	66.9	0.0000
5841010588180	40	59	76	59	44	61	56	61	108	14	9	22	23	632	82466	13	48.6	7.7	17.9	215.4	0.0000
5841010630855	51	48	78	70	113	109	75	64	77	111	78	68	133	1075	82466	13	82.7	13.0	5.5	66.4	0.0000
6605010370410	32	25	35	47	44	29	31	32	39	42	35	34	53	478	82466	13	36.8	5.8	1.0	12.4	0.4144
6605010954208	23	16	52	63	83	59	99	84	37	61	81	84	116	858	82466	13	66.0	10.4	7.4	88.5	0.0000
6620001487306	15	12	17	23	22	19	23	-9	-9	35	27	36	24	253	68690	11	23.0	3.7	1.3	12.9	0.2302
6710010033318	10	12	16	13	11	18	23	16	13	27	18	9	16	202	82466	13	15.5	2.4	1.6	19.7	0.0725
6710010200460	9	9	20	19	15	6	11	19	13	25	16	12	14	184	82466	13	14.5	2.3	1.8	21.7	0.0413
6710010200408	9	4	4	12	6	9	16	9	10	11	16	9	6	141	82466	13	10.8	1.7	5.6	67.6	0.0000
TOTAL	756	949	1017	1054	1075	1064	963	989	1117	1078	574	799	1011	12346	82466	13	949.7	149.7	37.9	454.5	0.0000
FLYING HOURS	4767	4519	5526	6480	6520	5975	6689	5918	6856	6444	6923	7083	7766								

NOTE -9 means missing data

NOTE -9 means missing data

## Appendix B

### THE VARIABILITY OF THE REPAIR PIPELINE FOR THE F-16

The figures in this appendix follow the same definitions and format given in the text for the F-15. Generally, the message contained in these data is much the same as for the F-15. Although the VTMRs of the pipelines are high, the F-16 does seem to do substantially better than the F-15 and the C-5 in terms of the ratio of the observed to expected number of parts in the repair pipeline for the important classes. During the time period when these data were collected, the contractor was still repairing many of the important F-16 avionics parts.

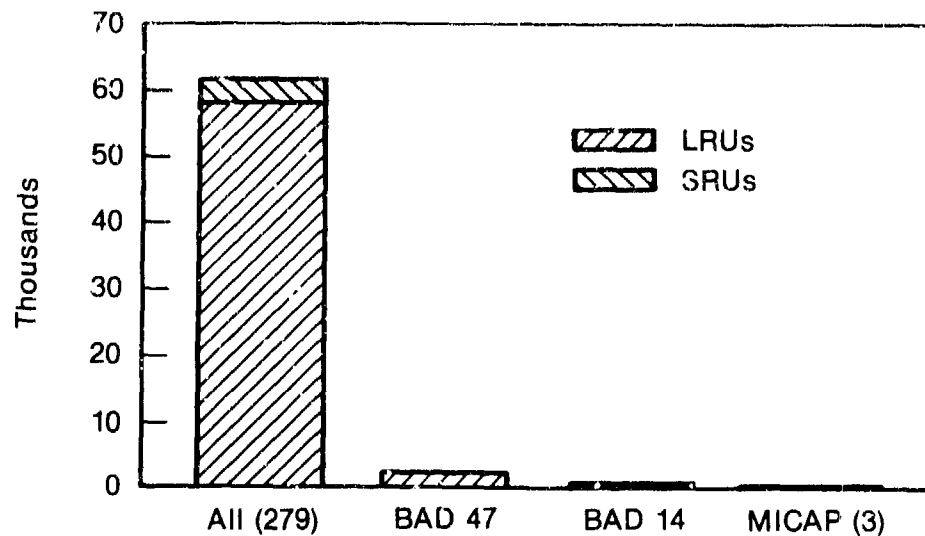


Fig. B.1—Number of F-16-peculiar parts in sample, by class

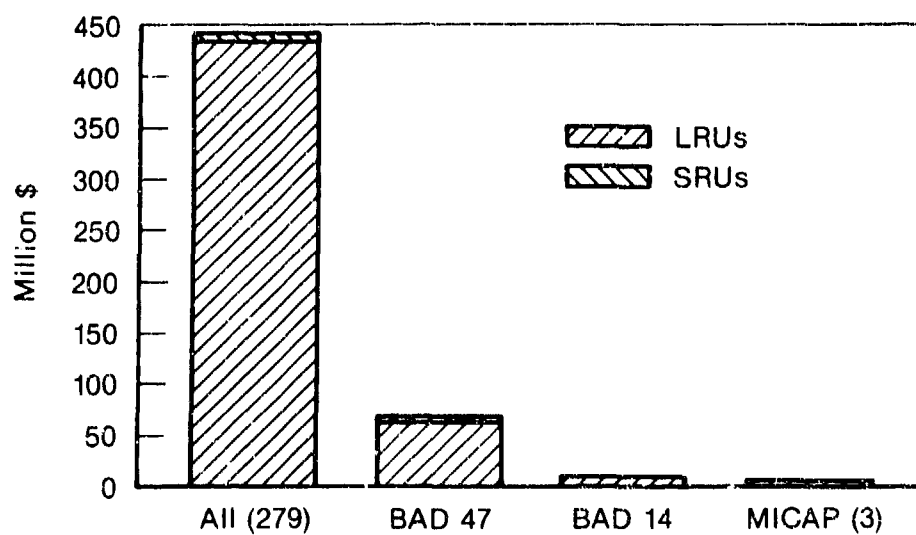


Fig. B.2—Cost of F-16-peculiar parts in sample, by class

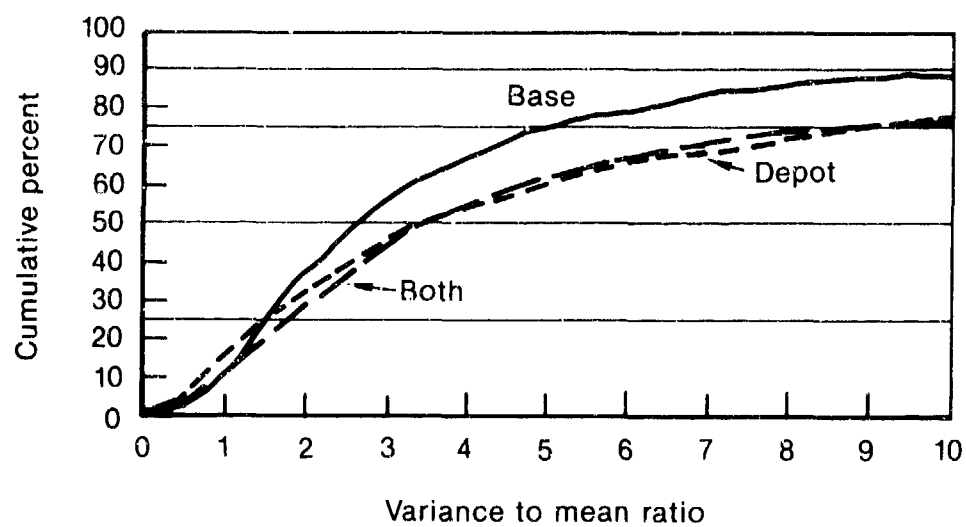


Fig. B.3—Cumulative F-16 VTMRs, all peculiar and common parts (1039)



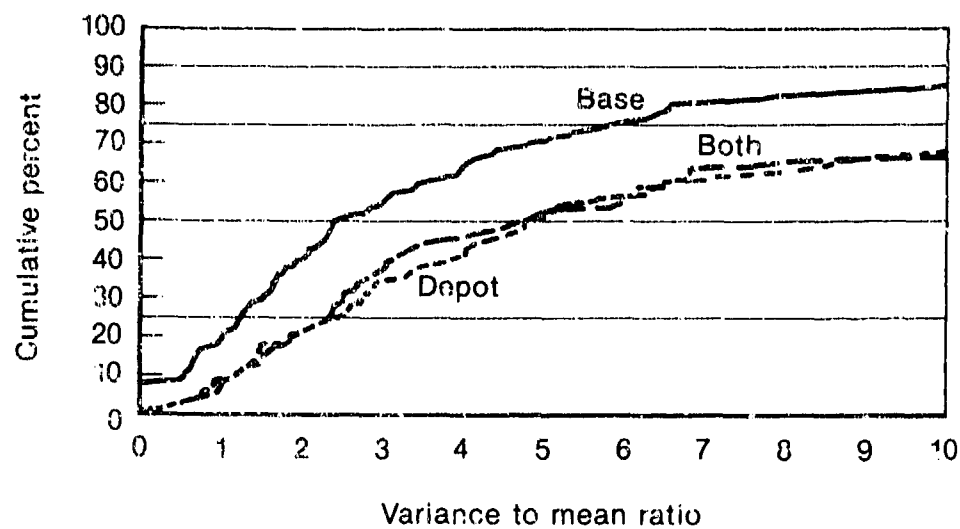


Fig. B.4--Cumulative VTMRs, F-16 BAD 139, peculiar and common parts

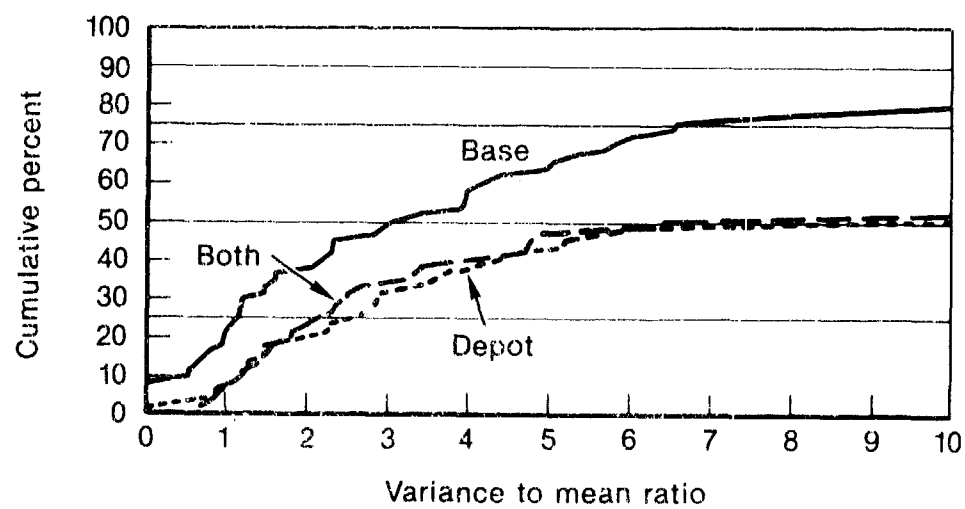


Fig. B.5--Cumulative VTMRs, F-16 BAD 59, peculiar and common parts

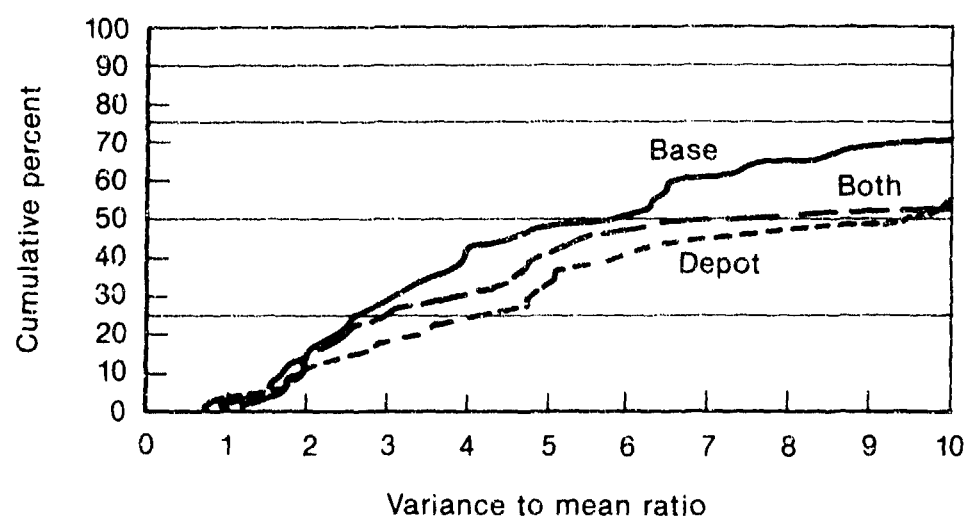


Fig. B.6—Cumulative VTMRs, F-16 MICAP (42),  
peculiar and common parts

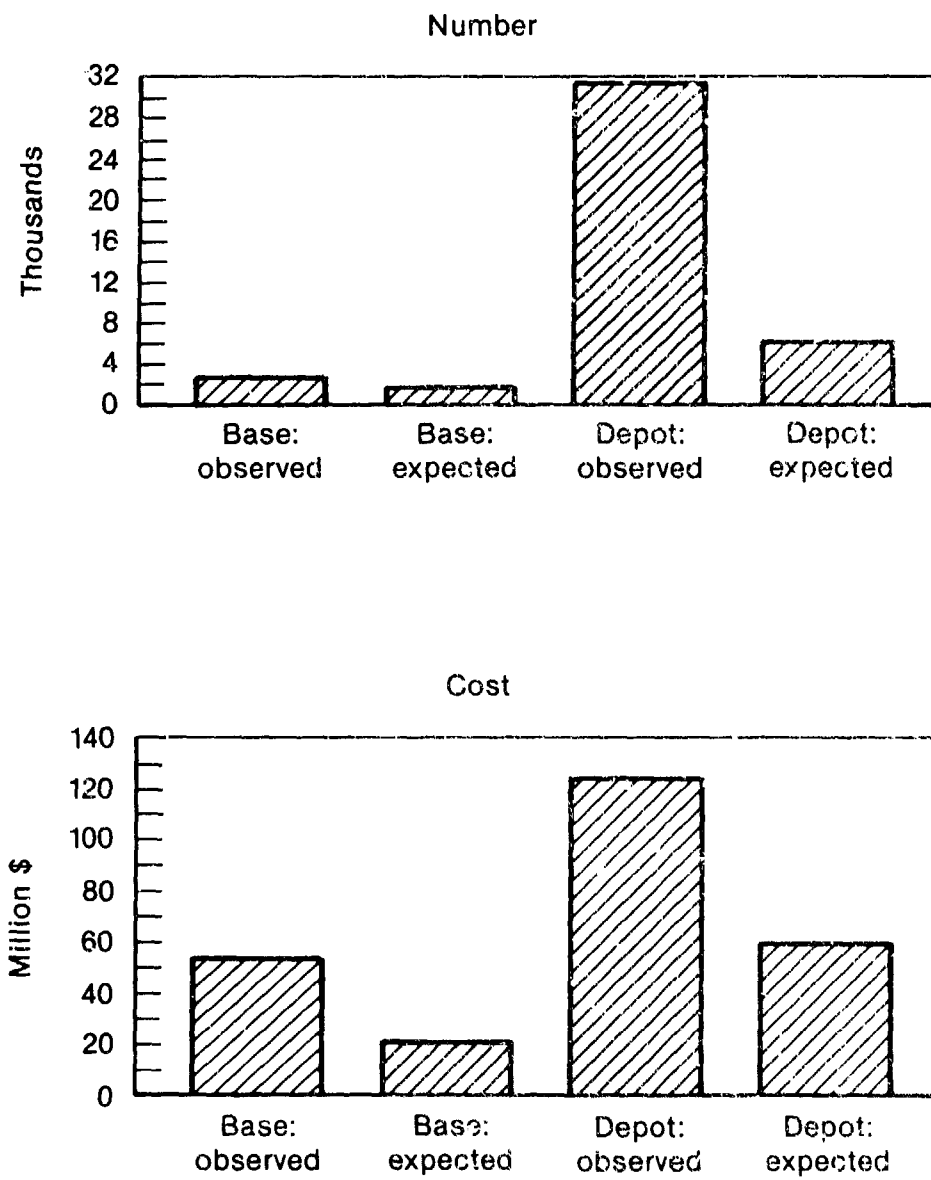


Fig. B.7—F-16: Number in and cost of LRU pipeline,  
all (278) peculiar parts

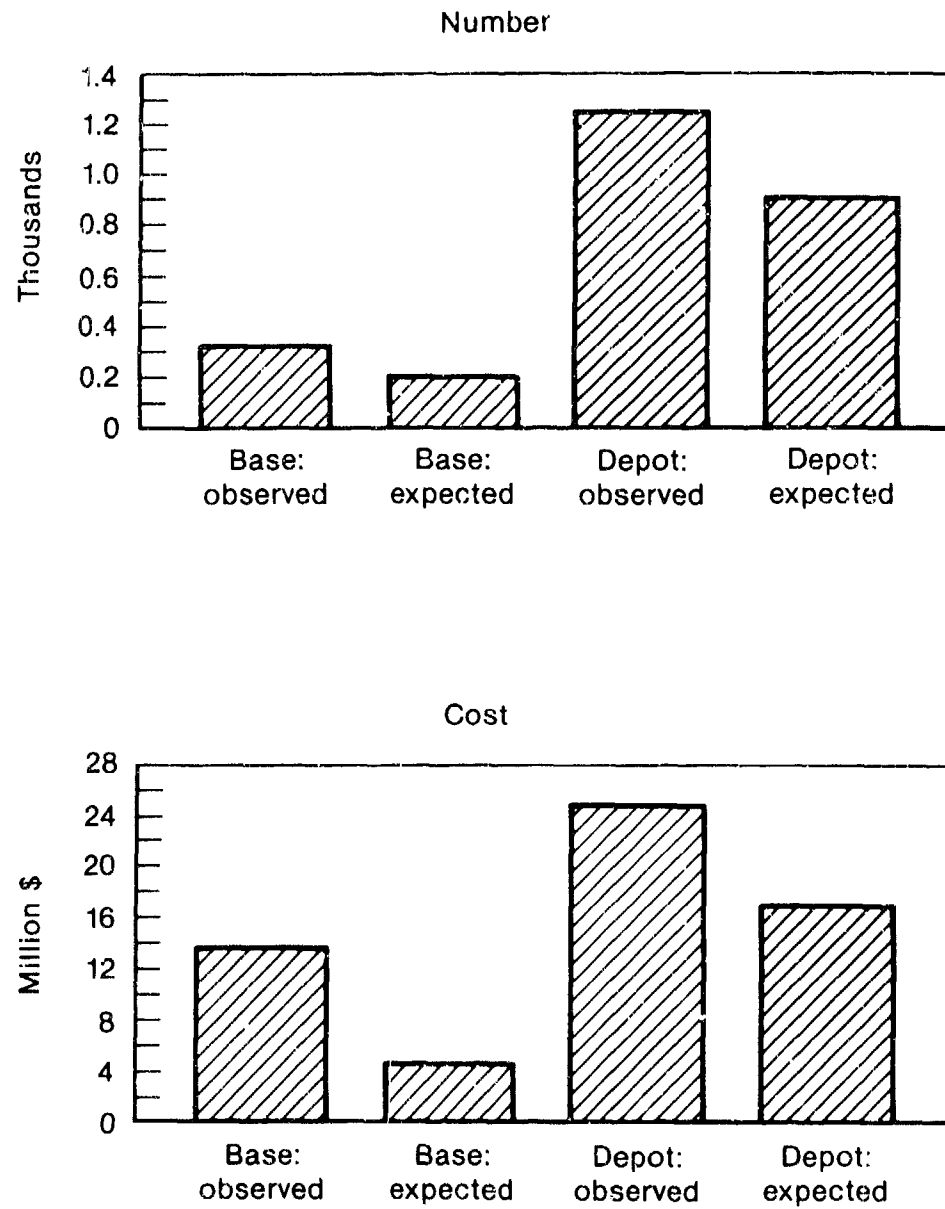


Fig. B.8—F-16: Number in and cost of LRU pipeline,  
critical 47 peculiar parts

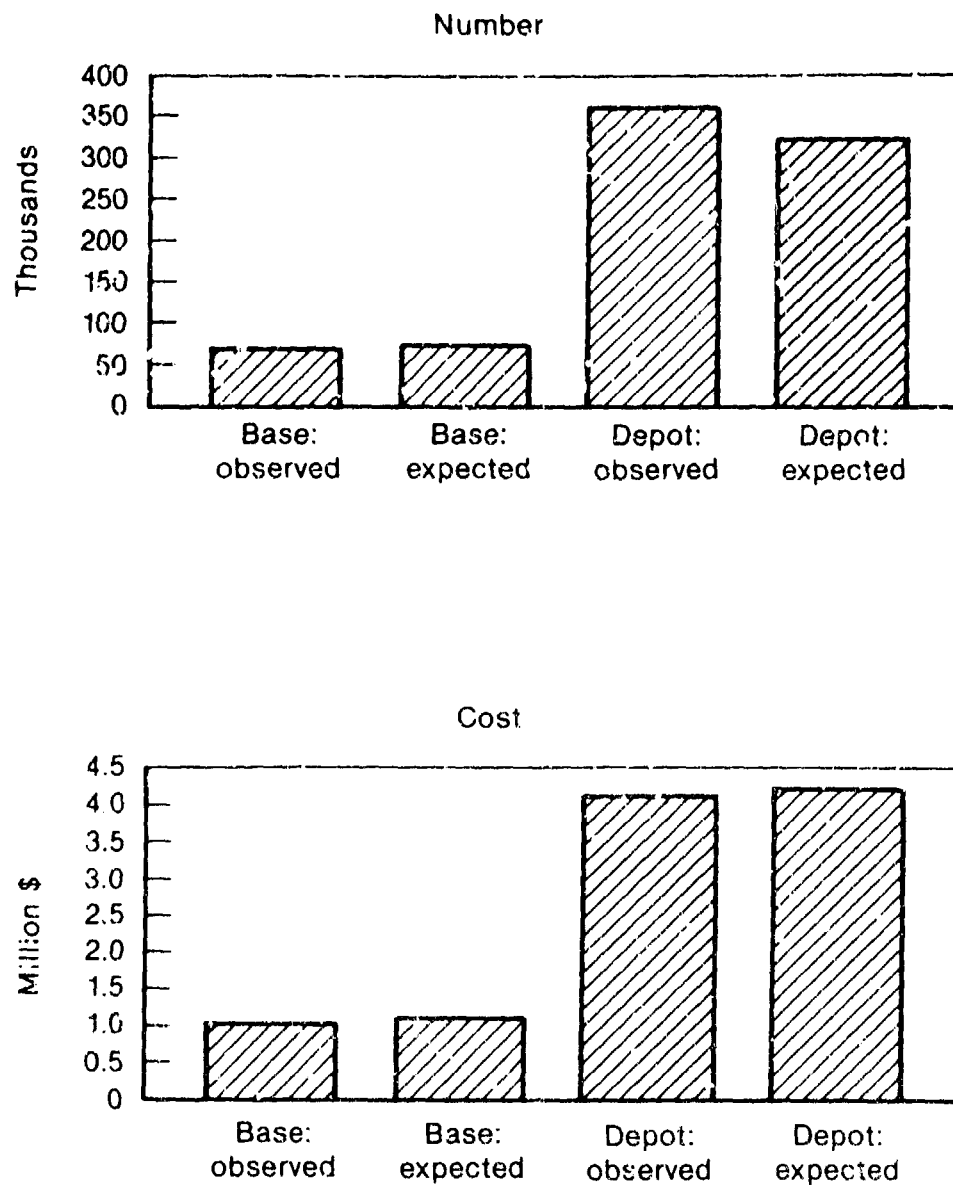


Fig. B.9—F-16: Number in and cost of LRU pipeline,  
critical 14 peculiar parts

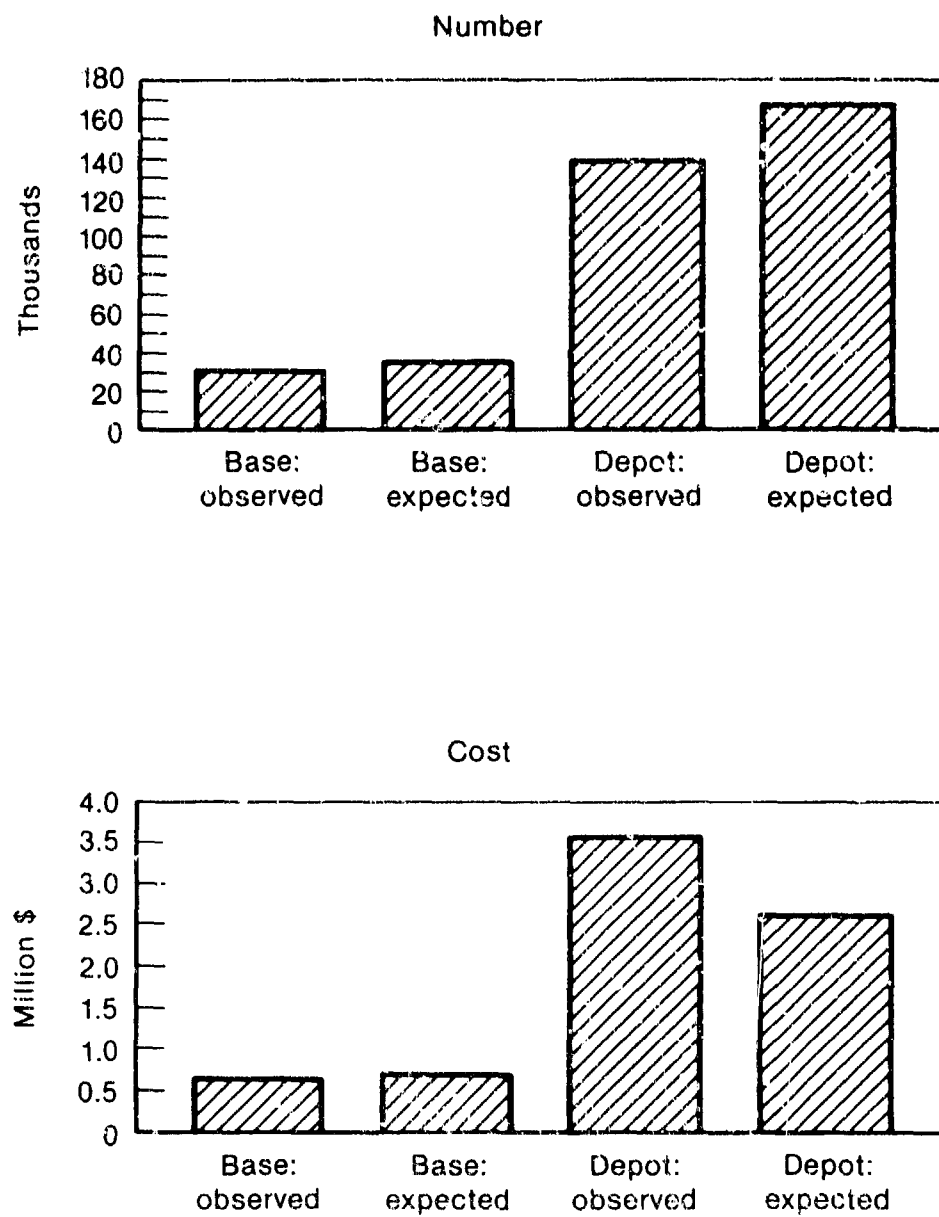


Fig. B.10—F-16: Number in and cost of LRU pipeline,  
MICAP (3) peculiar parts

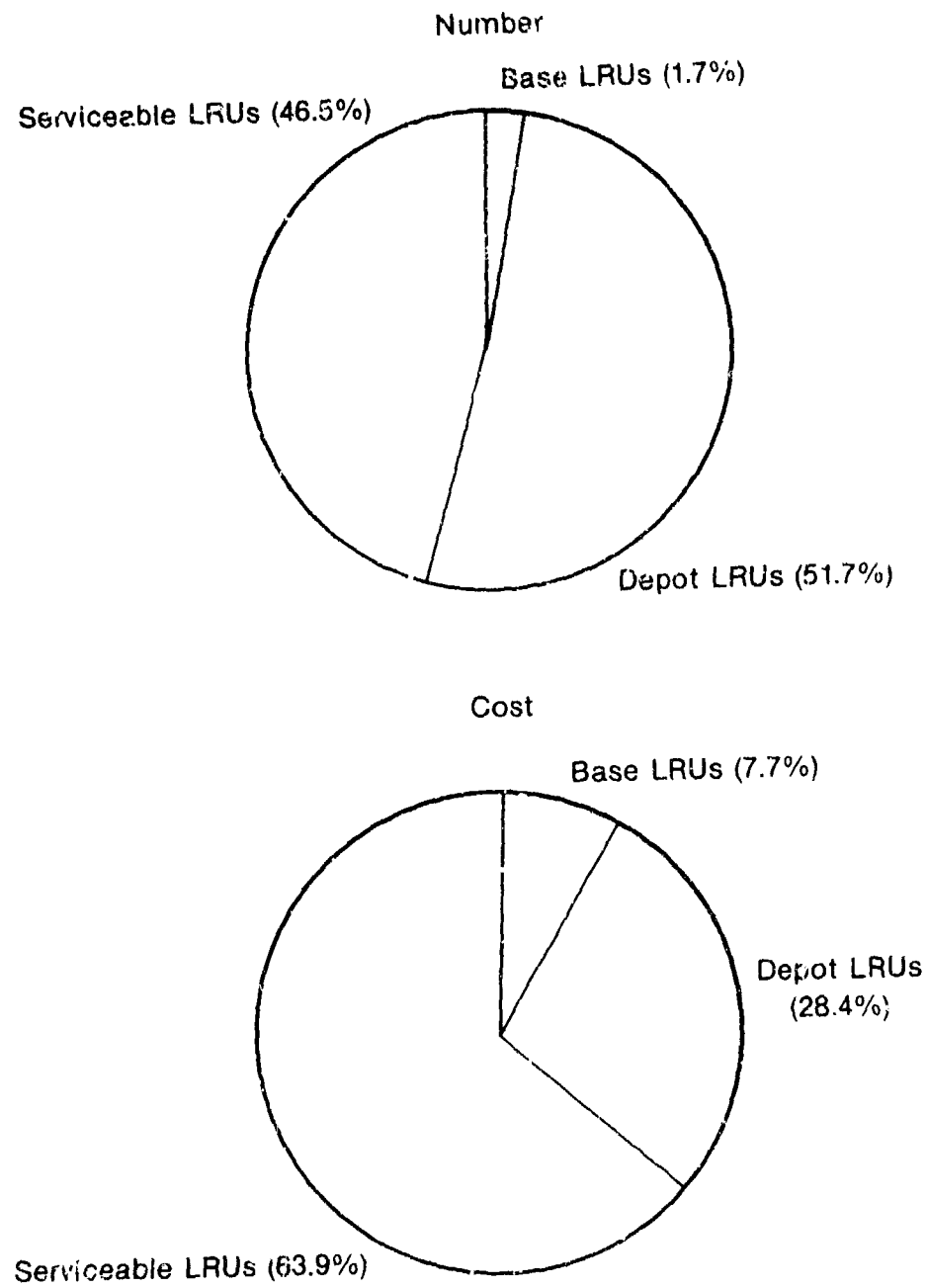


Fig. B.11—Serviceability of components, F-16 all (765)

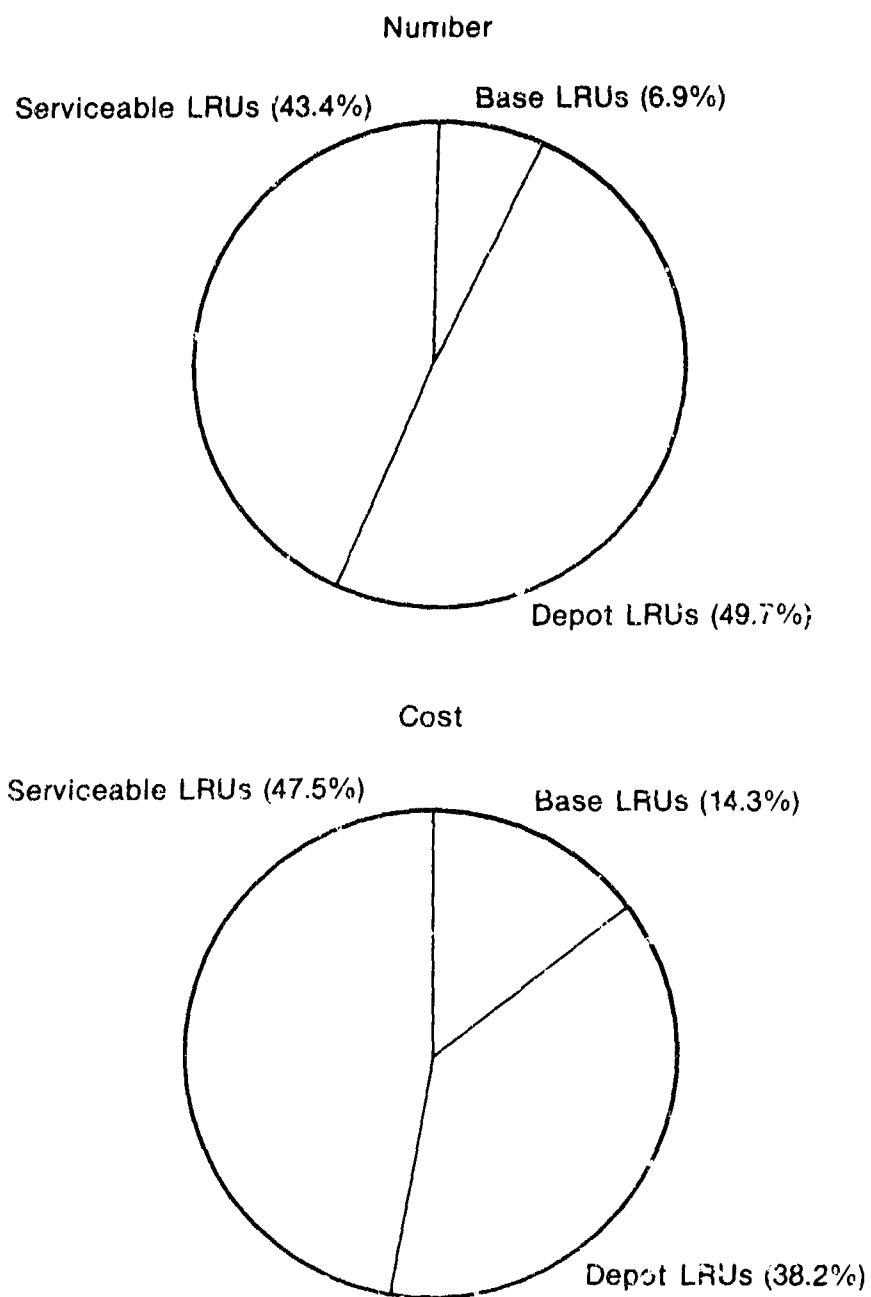


Fig. B.12—Serviceability of components, F-16 BAD 139



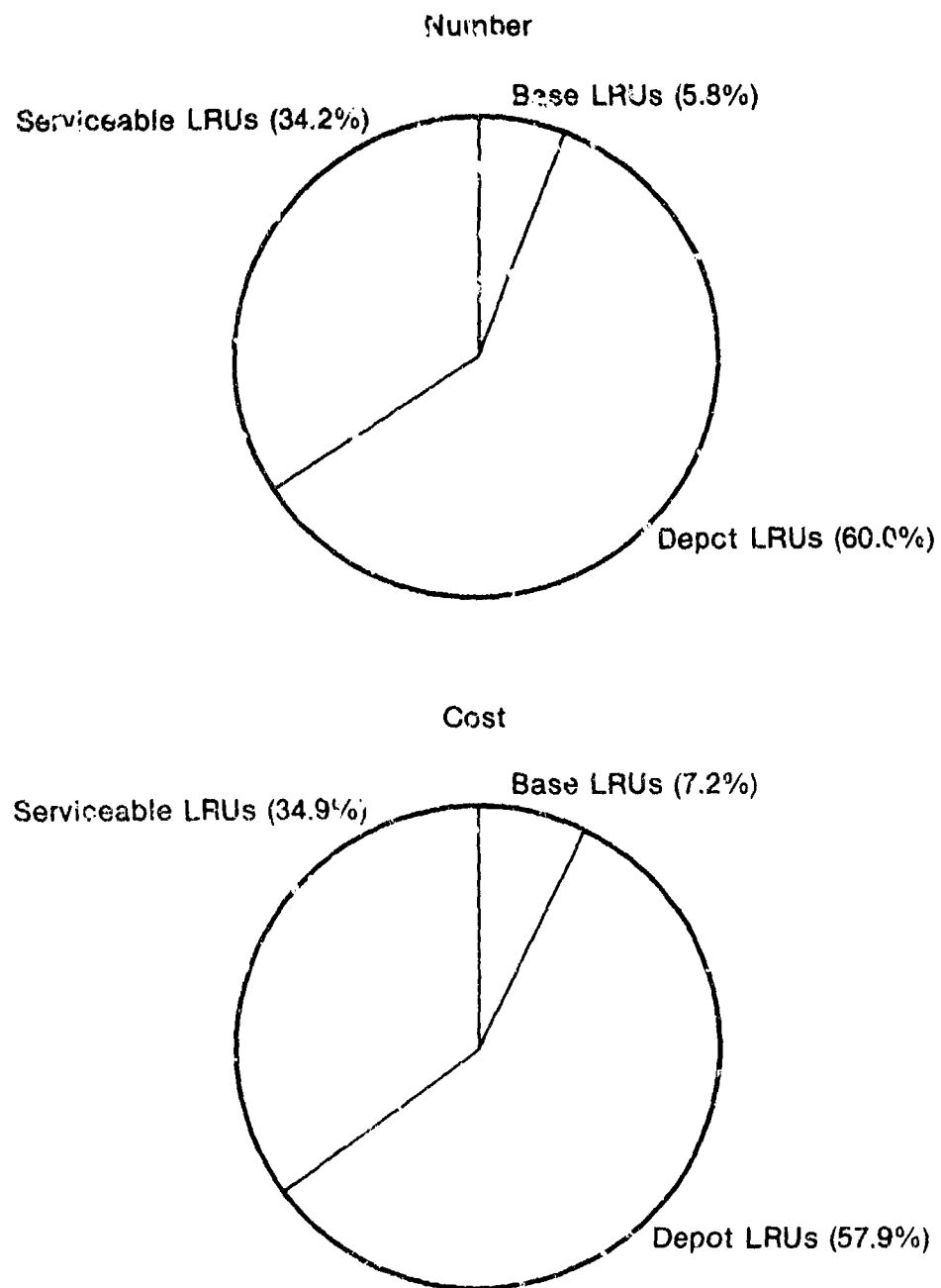


Fig. B.13—Serviceability of components, F-16 BAD 59

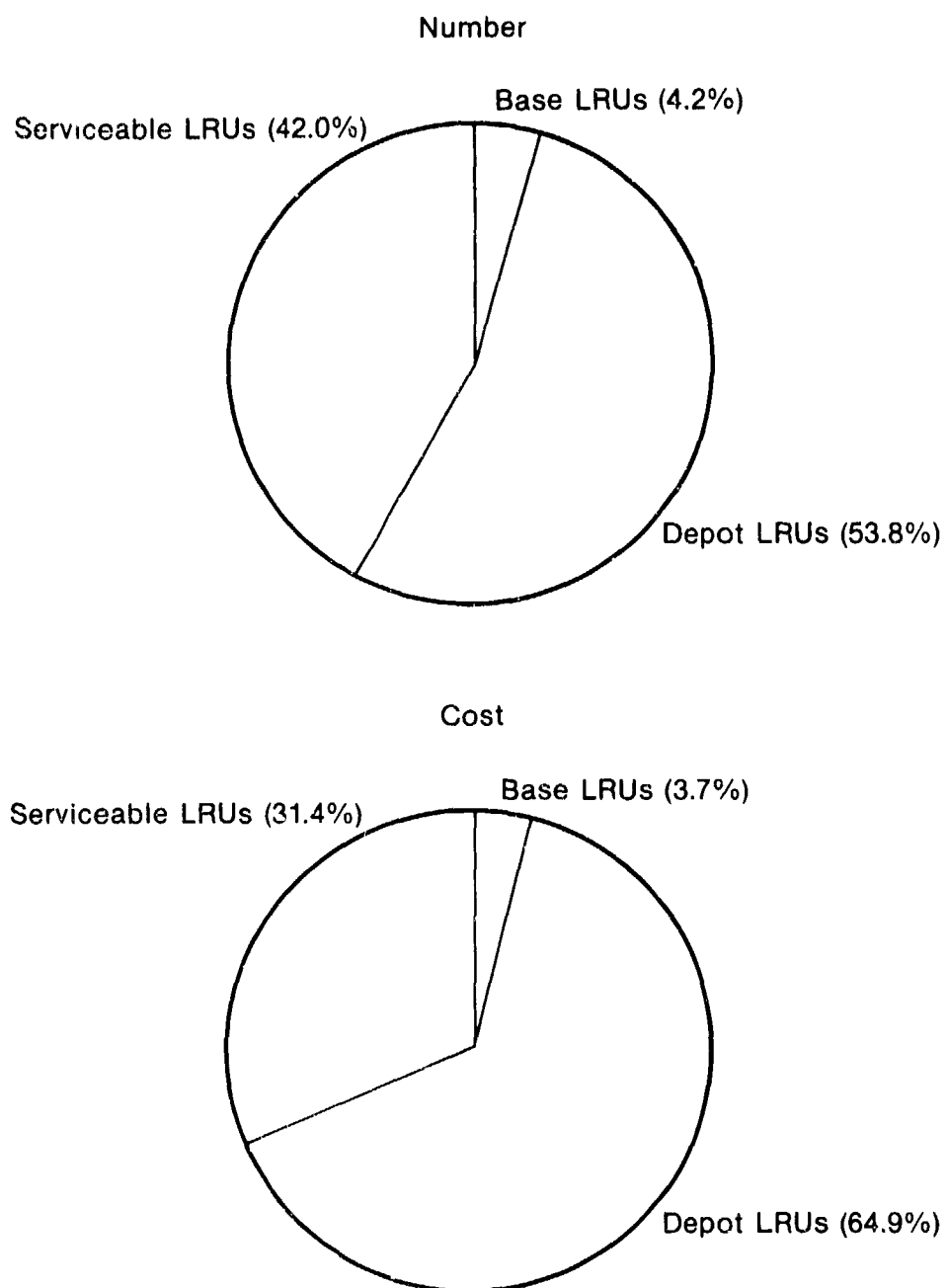


Fig. B.14—Serviceability of components, F-16 MICAP (42)

## Appendix C

### THE VARIABILITY OF THE REPAIR PIPELINE FOR THE C-5

The C-5 figures follow the format and definitions given in the text for the F-15. The message contained in these data are much the same as for the F-15: The VTMRs for the C-5 parts are high (although somewhat lower than for the F-15 and the F-16) and the ratio of observed to expected parts in the repair pipeline is higher even than for the other two weapon systems.

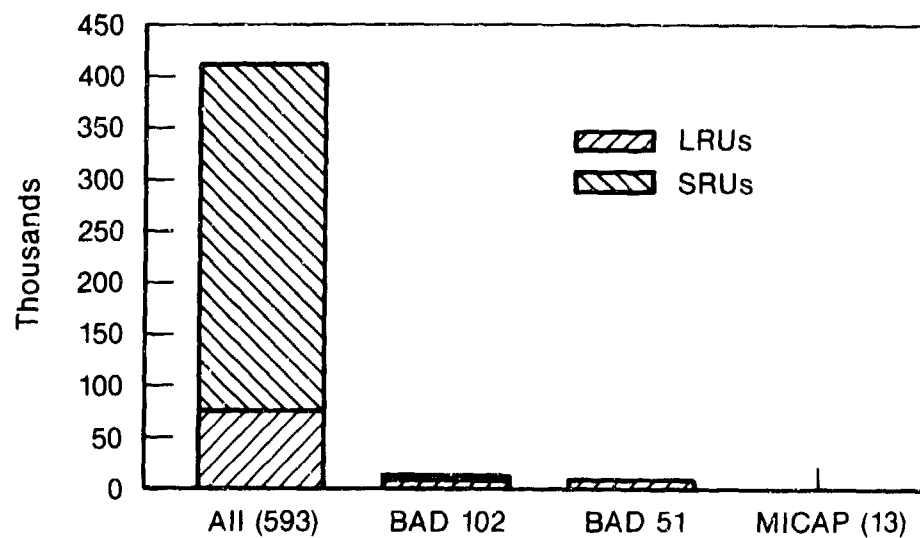


Fig. C.1—Number of C-5-peculiar parts in sample, by class

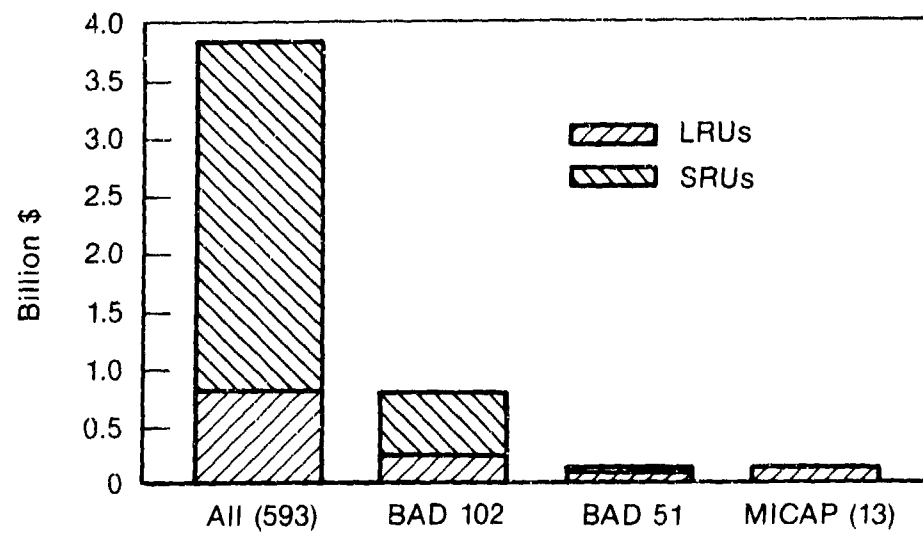


Fig. C.2—Cost of C-5-peculiar parts in sample, by class

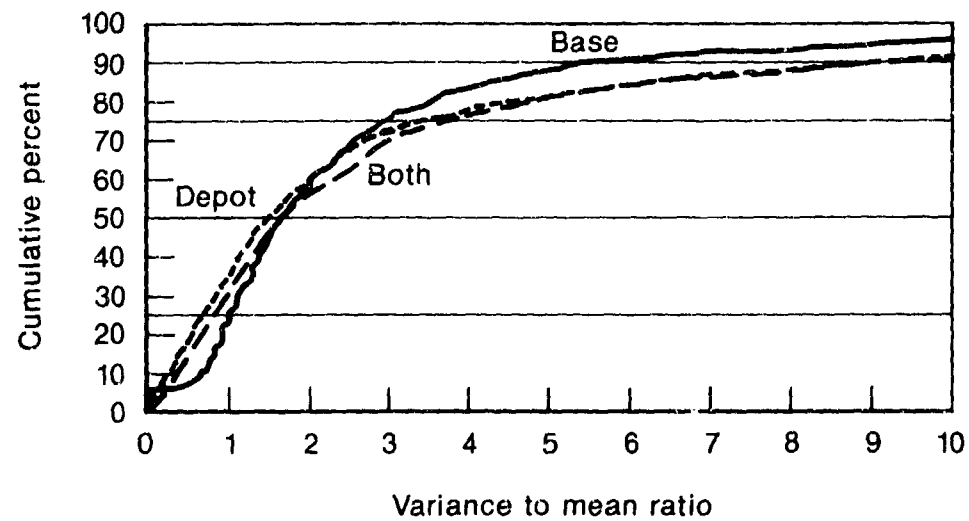


Fig. C.3—Cumulative C-5 VTMRs, all peculiar and common parts (833)

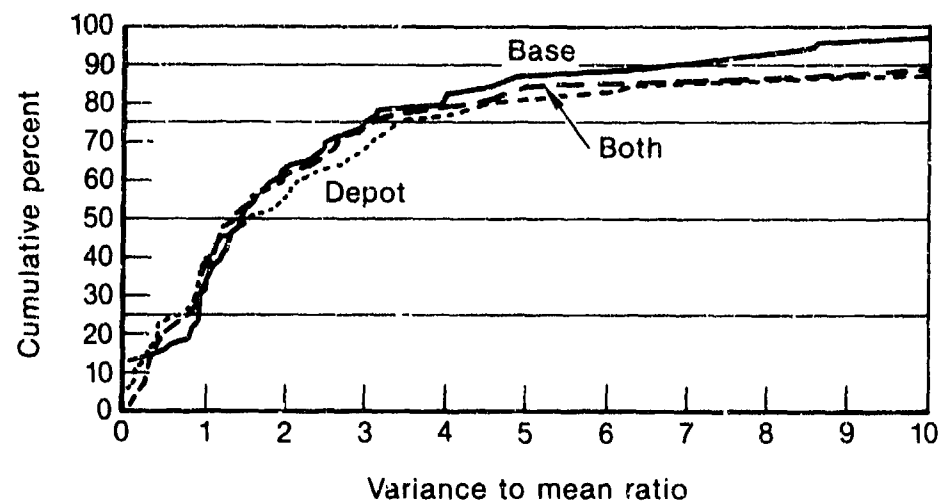


Fig. C.4—Cumulative C-5 VTMRs, all peculiar and common parts,  
BAD 149

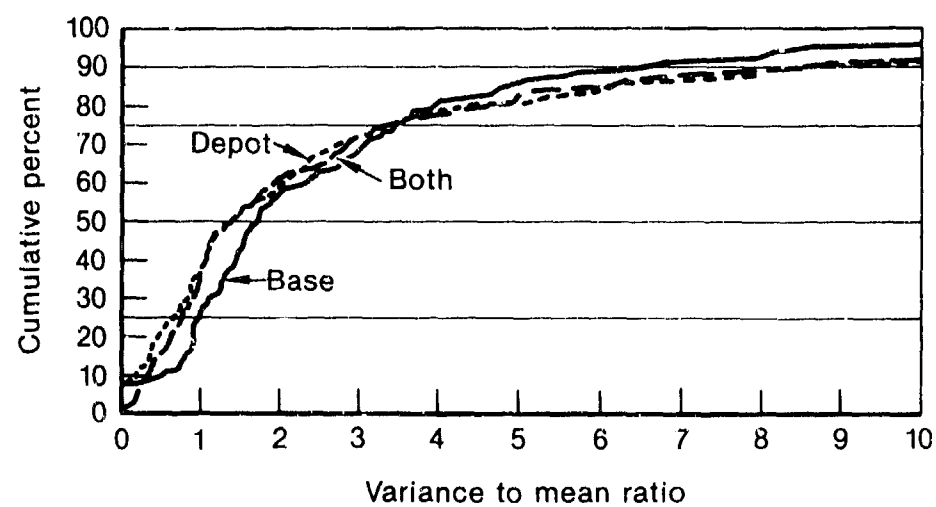


Fig. C.5—Cumulative C-5 VTMRs, all peculiar and common parts,  
BAD 68

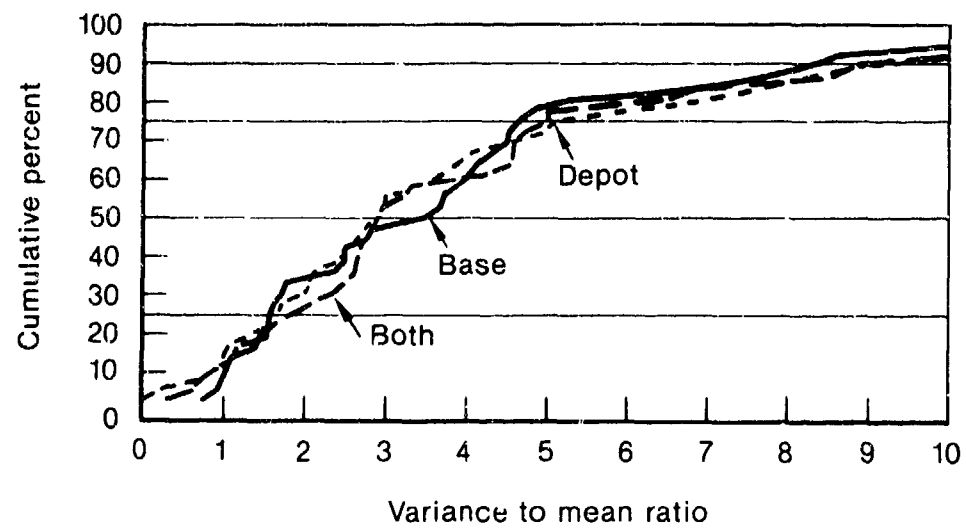


Fig. C.6—Cumulative C-5 VTMRs, all peculiar and common parts, MICAP (35)

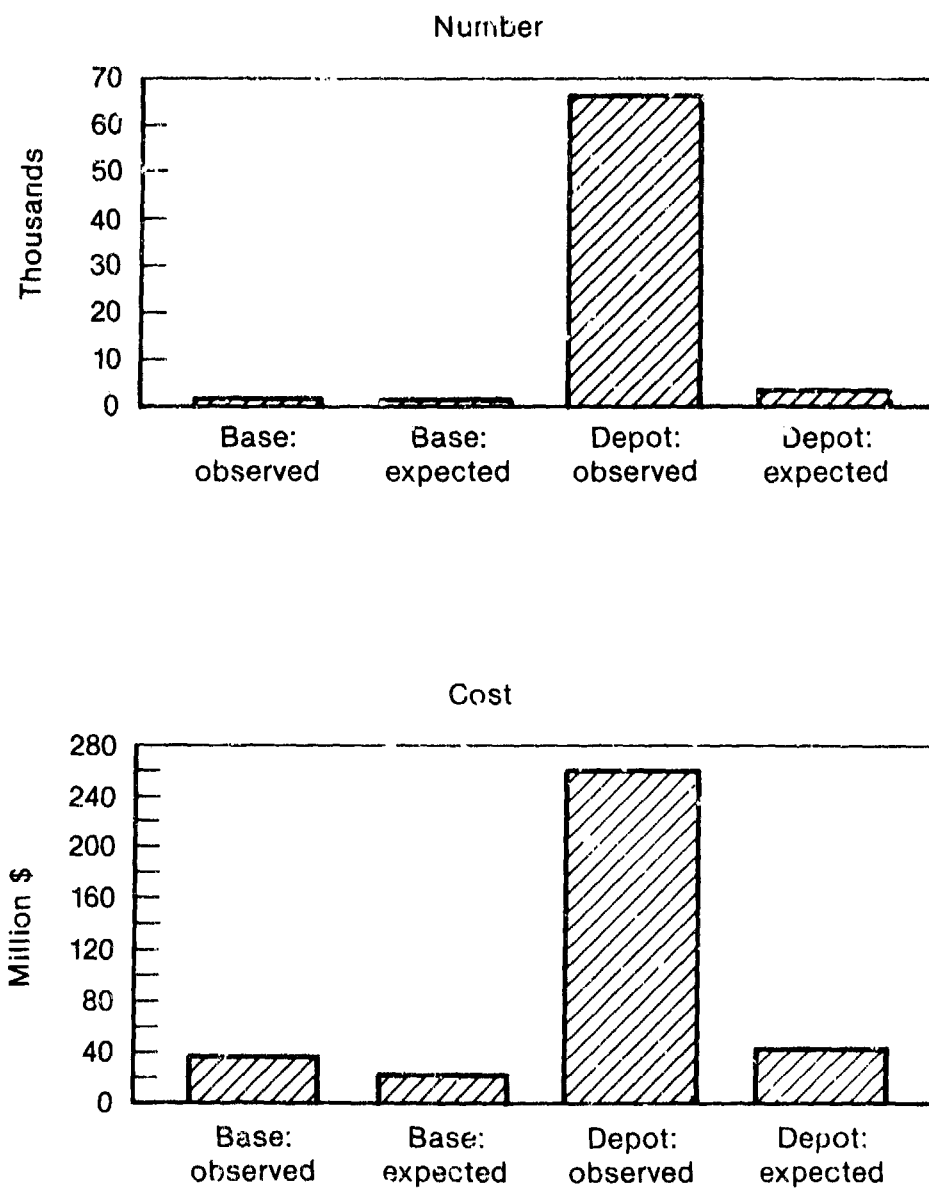


Fig. C.7—C-5: Number in and cost of LRU pipeline,  
all (583) peculiar parts

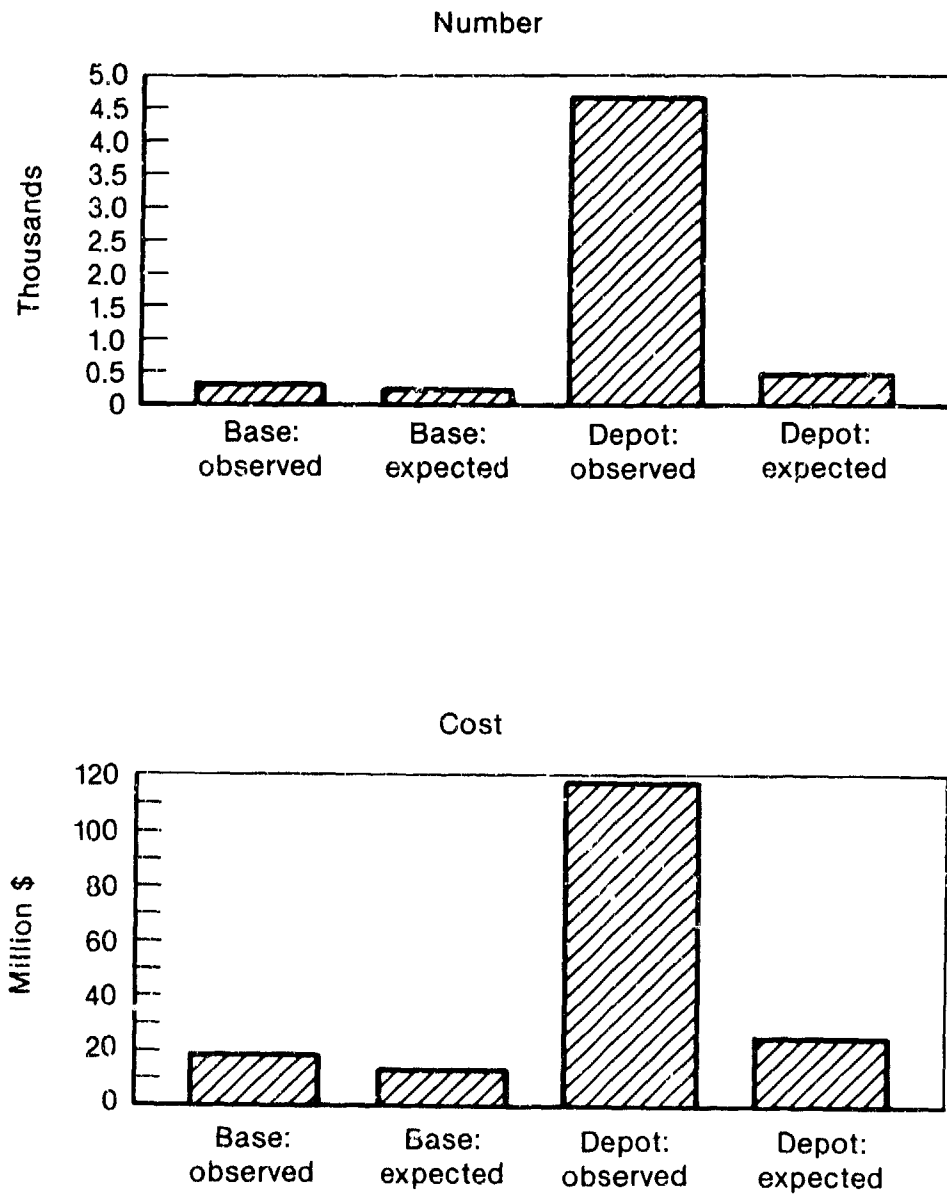


Fig. C.8—C-5: Number in and cost of LRU pipeline,  
critical 102 peculiar parts



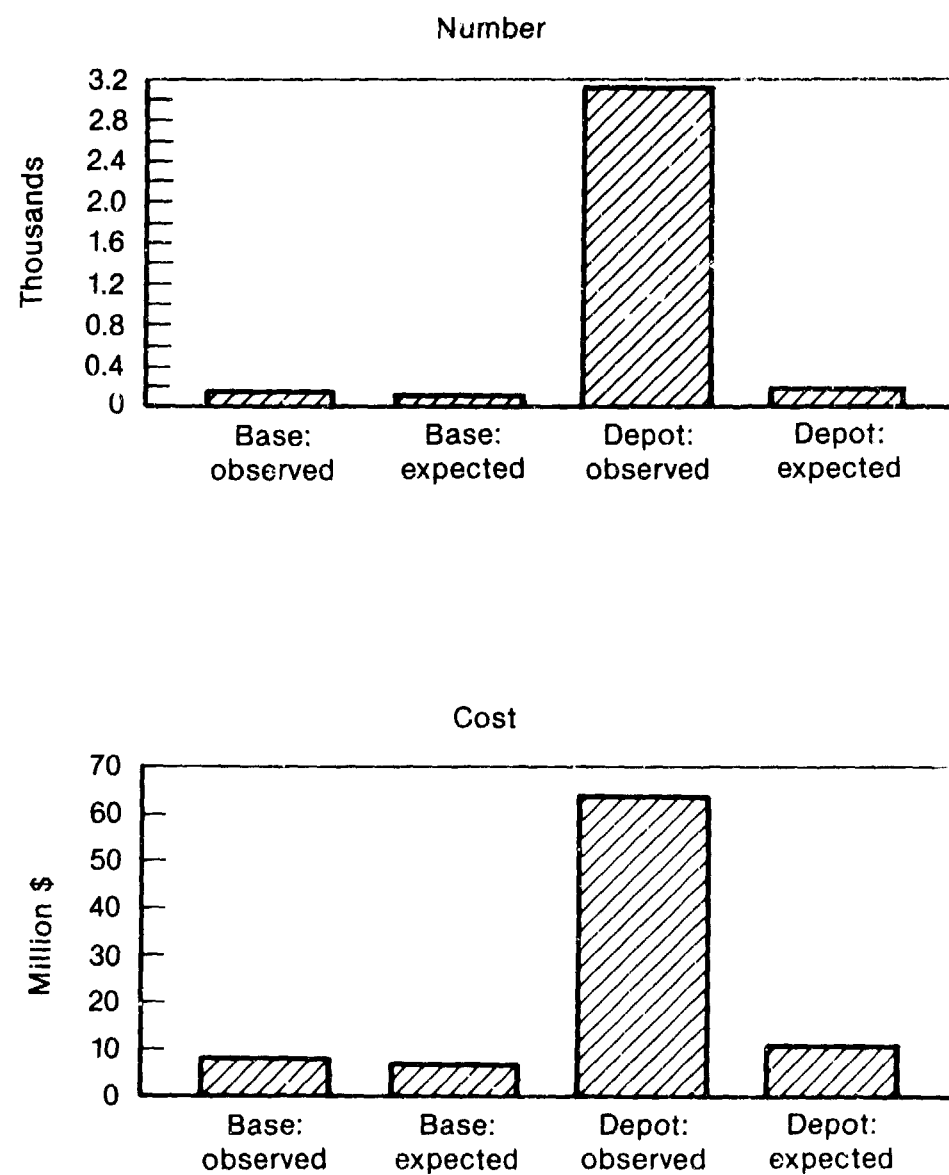


Fig. C.9—C-5: Number in and cost of LRU pipeline,  
critical 51 peculiar parts

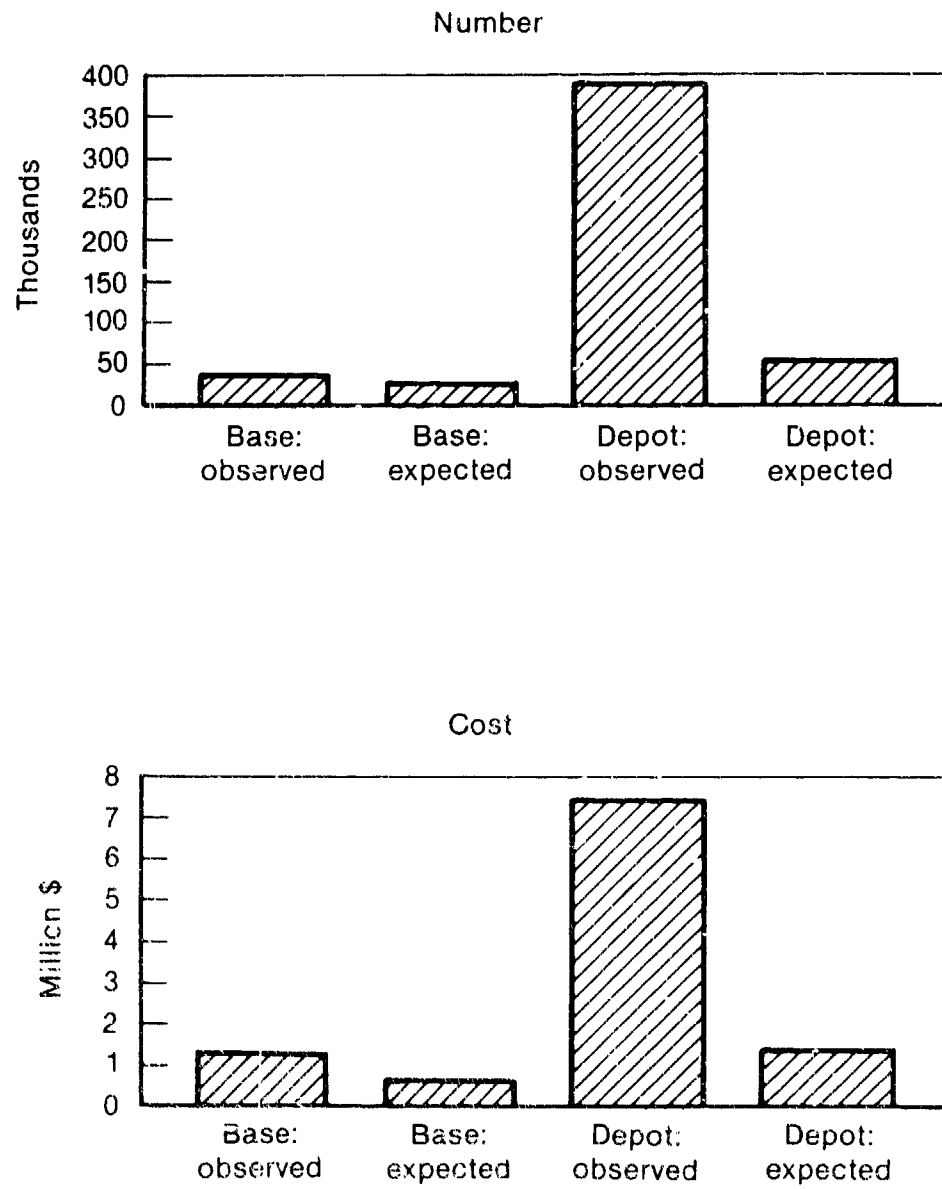


Fig. C.10--C-5: Number in and cost of LRU pipeline,  
MICAP (13) peculiar parts

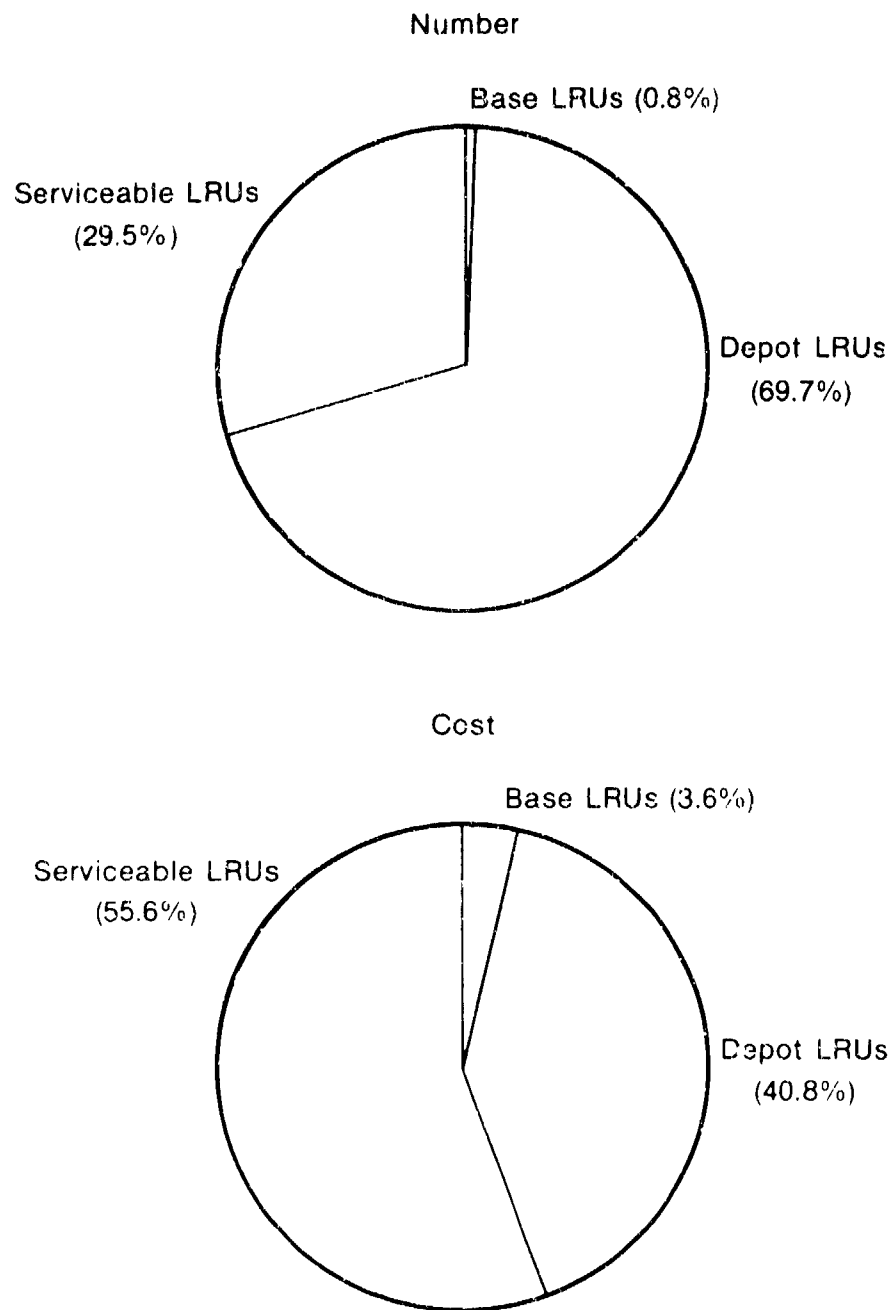


Fig. C.11—Serviceability of components, C-5 all (833)

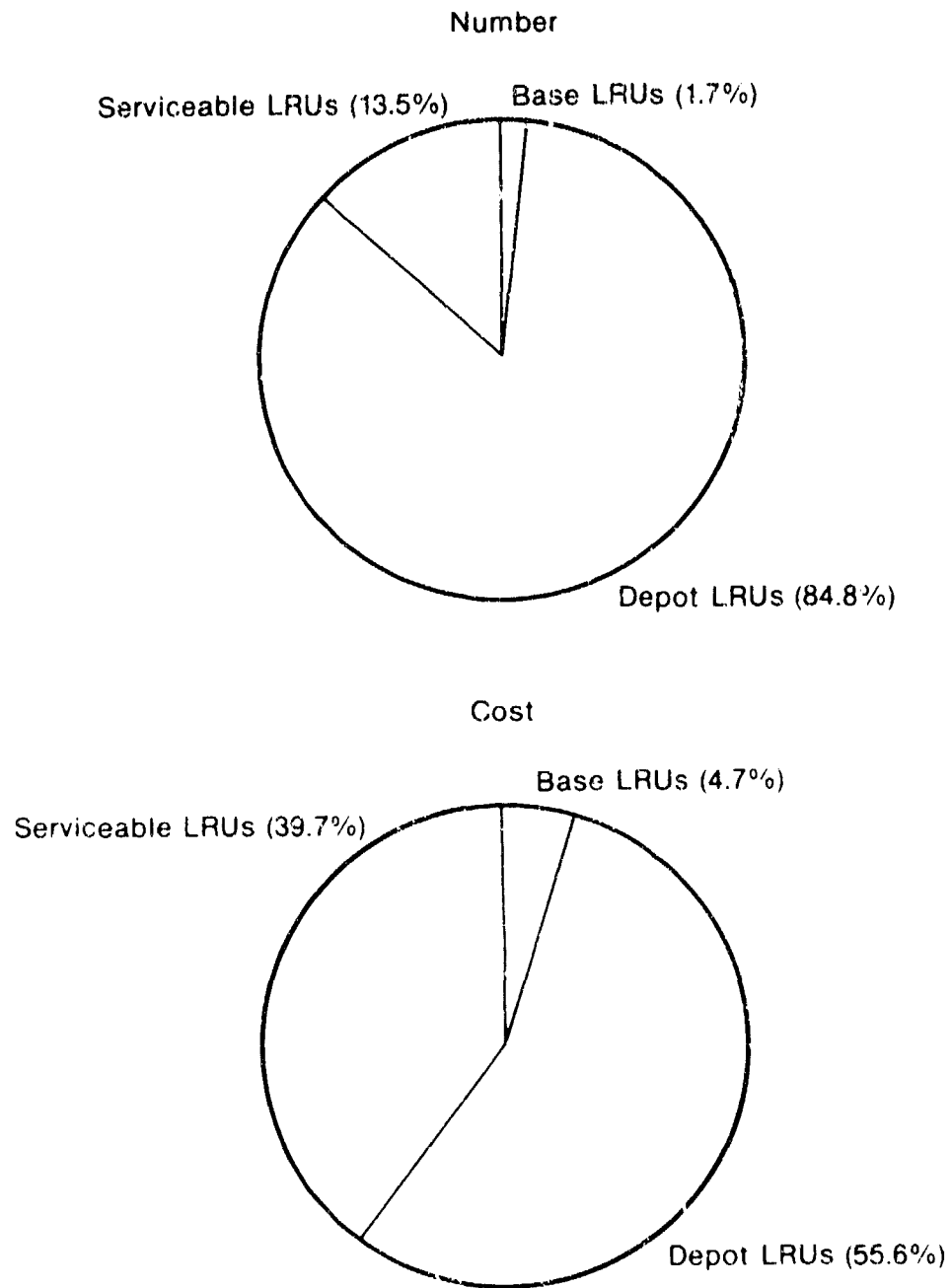


Fig. C.12—Serviceability of components, C-5 BAD 149

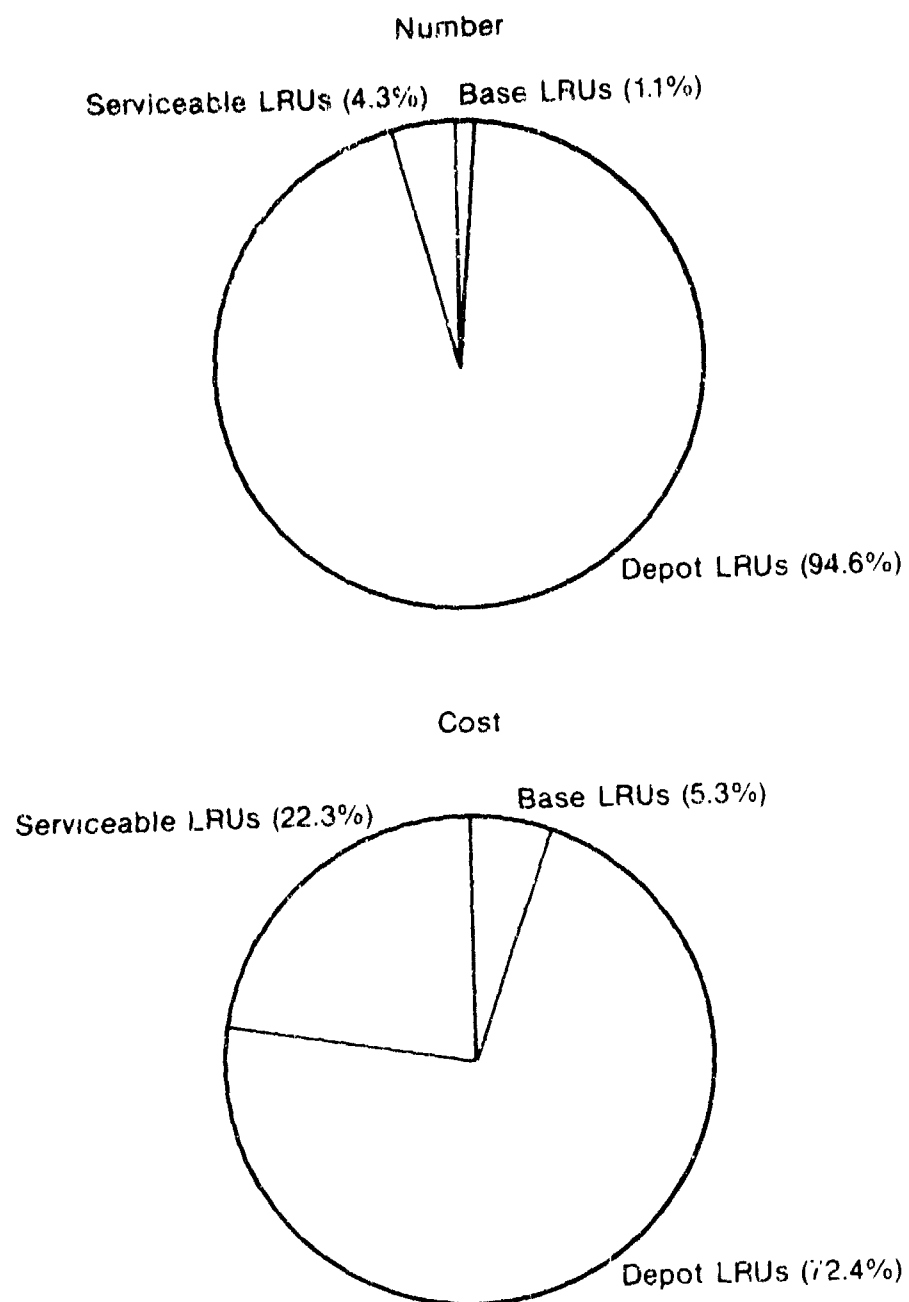


Fig. C.13--Serviceability of components, C-5 BAD 68

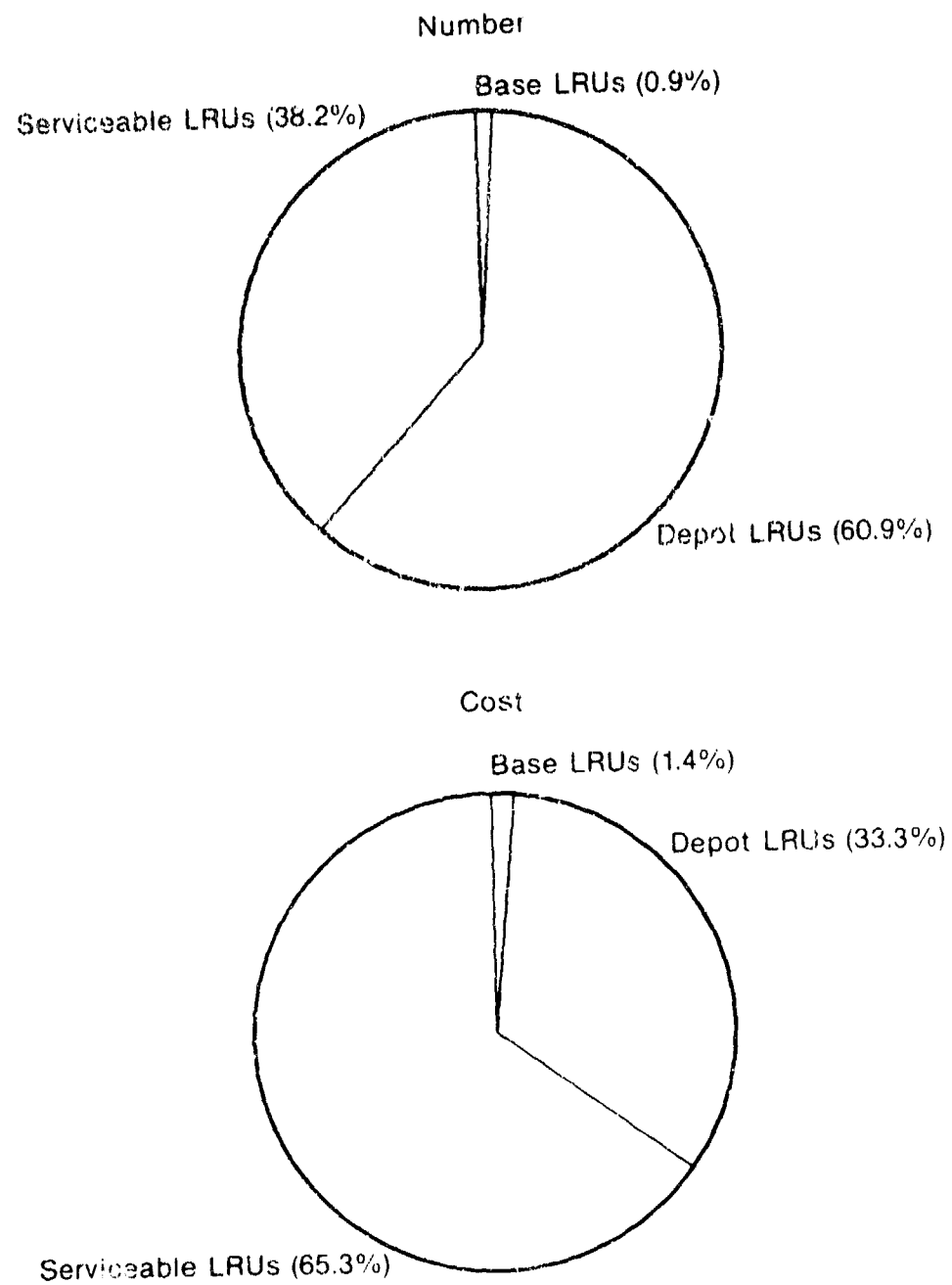


Fig. C.14—Serviceability of components, C-5 MICAP (35)

## Appendix D

### ESTIMATING THE VARIANCE-TO-MEAN RATIO

The equations below are taken from James S. Hodges, *Modeling the Demand for Spare Parts: Estimating the Variance-to-Mean Ratio and Other Issues*, The RAND Corporation, N-2086-AF, May 1985. These formulas have been used throughout the text of this report and are included here for completeness.

As the underlying distributions depart from the simple Poisson, the ability to accurately estimate the relevant parameters of the distributions quickly degrades. As a result, the meaningful use of these estimates is fraught with difficulties.

The reader is referred to N-2086-AF for an excellent discussion of the properties of these estimates.

Let

$X_i$  = number of demands in period  $i$ ,  $i = 1, 2, \dots, n$

$N = \sum X_i$  = total number of demands over all  $n$  periods

$f_i$  = number of flying hours in period  $i$ ,  $i = 1, 2, \dots, n$ , and

$T = \sum f_i$  = total number of flying hours in all periods.

If  $N > 0$ , our estimate  $\rho$  of the variance-to-mean ratio is defined to be

$$\rho = S^2/\lambda',$$

where  $\lambda' = N/T$  and

$$(n-1)S^2 = \sum (X_i - f_i\lambda')^2 / f_i = \sum f_i (X_i/f_i - \lambda')^2.$$

If  $N = 0$ , both the numerator and denominator of  $\rho$  are zero, and  $\rho$  is typically defined to be 1.

## Appendix E

### POISSON DENSITIES AND ARRIVAL PROCESSES

This appendix defines some of the terms used elsewhere in the report.

#### THE BINOMIAL DISTRIBUTION

The binomial distribution has density  $B(m|n,p)$ ,  $m = 0,1,\dots,n$ ,

$$B(m|n,p) = C(n,m)p^m(1-p)^{(n-m)},$$

where  $0 < p < 1$ , and  $C(n,m) = n!/m!(n-m)!$  is, for integer  $n$  and  $m$ , the number of combinations of  $n$  things taken  $m$  at a time.

If  $n$  independent sorties were flown, each giving rise to a certain failure with probability  $p$ , then the probability of exactly  $m$  failures would be given by  $B(m|n,p)$ .

It follows from the form of the density that a binomial random variable  $X$  has expectation  $E(X) = np$ , and variance  $\text{Var}(X) = np(1-p)$ ; hence the VTMR( $X$ ) =  $1-p$  is always less than one. If  $p$  goes to zero and  $n$  goes to infinity in such a way that  $E(X)$  converges to a finite limit, then  $B(m|n,p)$  converges to the Poisson density (see below) with mean  $E(X)$  and VTMR = 1 (Feller, 1968). In this sense the Poisson distribution is a limiting case of the binomial distribution.

Similarly, if the probability of failure is not constant over sorties or different aircraft—if the  $i$ th sortie has a failure with probability  $p_i$ , then, assuming independent sorties, the VTMR of the number of failures is equal to  $\sum p_i(1-p_i)/\sum p_i = 1 - \sum p_i^2/\sum p_i$ , which is never greater than one.<sup>1</sup>

It can also be shown with a characteristic function argument that if the  $p_i$  uniformly approach 0 as  $n$  gets large and  $E(X)$  converges, then the number of failures is asymptotically a simple Poisson random variable, as would be the case if all the  $p_i$  are equal.

<sup>1</sup>There has been a recurring effort to show that high VTMRs are a result of the differences between aircraft. As extensions of this simple argument show, these efforts are not likely to be fruitful.



### THE SIMPLE POISSON DISTRIBUTION

The simple Poisson distribution has density  $P(m|c)$ ,  $m = 0, 1, 2, \dots$ ,

$$P(m|c) = \exp(-c)c^m/m!$$

It follows that a simple Poisson random variable  $X$  has mean and variance

$$E(X) = c$$

$$\text{Var}(X) = c$$

and hence has VTMR = 1.

### THE (STATIONARY) SIMPLE POISSON ARRIVAL PROCESS

If the random variable  $X$  counts the number of occurrences up to time  $t$  of an event of interest, such as the failures of an aircraft component, we denote the functional dependence of  $X$  on time by  $X(t)$ . In that case we define  $E(X(t)) = H(t)$ . An arrival process is a counting process:  $X(t)$  is a nondecreasing, nonnegative integer valued random function. It follows that  $H(t)$  is also nondecreasing in  $t$ .

If failures of a certain aircraft component were such that the time between events had an exponential distribution with mean  $1/a$ , then the number  $X$  of such events that occurred in a time interval of length  $t$  would be a simple Poisson random variable with mean  $c = at$ . (In fact  $X$  will have a simple Poisson distribution if, and only if, the inter-arrival times have an exponential distribution; see Feller, 1968.) In this case all time intervals of equal length are equally likely to contain an observation. Such processes are said to be stationary in time, and  $H(t)$  has the simple form  $H(t) = at$  for some constant  $a$ .

### THE NONSTATIONARY SIMPLE POISSON PROCESS

In most practical applications it is assumed that the distribution of the number of observations in a time interval  $(t, t + \delta]$  depends only on  $\delta$ , the length of the interval. Typically this approximation is at best accurate only in the short run. It is often preferable to assume that the probability of an arrival is asymptotically equal to  $h(t) \times \delta$ , where  $h(t)$  is typically assumed to be known, at least to within multiplicative constants. It follows that  $E(X(t)) = H(t)$ , where  $H(t)$  is the integral to

t of  $h(s)ds$ . If it is assumed that failures are proportional to flying hours,  $h(t)$  represents the number of aircraft flying at time  $t$ , and  $H(t)$  is cumulative flying hours. Many of the applicable properties of stationary Poisson processes carry over to the nonstationary Poisson process.

If  $X(t)$  is a nonstationary Poisson process, the number of observations in an interval  $(a,b]$  is a Poisson random variable with mean  $H(b) - H(a)$ .

### THE COMPOUND POISSON PROCESS

Suppose that *events* occur as above, hence the number of events in any time interval  $(a,b]$  is a Poisson random variable with mean  $H(b) - H(a)$ . But suppose that instead of an event being a single failure of a component, an event comprises a cluster of failures, where the probability the cluster is of size  $n$  is given by a density function  $f(n)$ . Then the number of failures is said to have a compound Poisson distribution. The simple Poisson is a special case, where  $f(1) = 1$ , and  $f(n) = 0$  for all  $n$  not equal to one. If  $f(1) = p$  and  $f(0) = 1 - p$ , then the resulting process is a "censored" Poisson process (Feller, 1968, p. 160) and as such is a simple Poisson process with VTMR equal to 1. Nontrivial compound processes occur when the compounding density has mass at integers greater than 1.

Regardless of the choice of  $f$ , the VTMR of a compound Poisson random variable  $X$  is a function of the compounding density  $f$  only:

Let  $U$  be the number of arrivals in a cluster for the compound Poisson arrival process, and let  $N$  be the number of clusters. Then,

$$E(X) = E(N)E(U)$$

and

$$\text{Var}(X) = E[\text{Var}(X | N)] + \text{Var}(E(X | N))$$

$$= E[N\text{Var}(U)] + \text{Var}[NE(U)]$$

$$= E(N)[\text{Var}(U) + E(U)^2]$$

It follows that the VTMR of  $X$  is given by

$$\text{Var}(X)/E(X) = E(U) + \text{Var}(U)/E(U) ,$$

which clearly depends only on the first and second moments of  $U$  and hence is a function of the compounding density  $f$ , but does not depend on  $H$ .<sup>2</sup>

Since cluster size is an integer valued random variable, it follows that  $\text{Var}(U)$  is greater than  $E(U)$  unless  $f$  has all of its mass at 0 and 1 (in which case the "compounding" distribution is in fact a censoring distribution, and  $\text{Var}(X)/E(X) = 1$ ), hence the VTMR of a compound Poisson distribution is greater than one.

### A MIXTURE OF (SIMPLE) POISSON DISTRIBUTIONS

Suppose that  $X$ , the number of failures of an aircraft component, is known to have a stationary simple Poisson distribution with mean  $c$ , but  $c$  is itself a random variable having some known density  $g(c)$ . Then the distribution of  $X$  is a mixture of Poisson distributions. Mixed Poisson distributions are useful in describing the situation where we believe  $X$  is a simple Poisson random variable, and there is reason to assume that its mean is also a random variable. If the density  $g(c)$  puts all its mass at some one value of  $c$ , then we know the mean and the distribution of  $X$  with certainty—it is simple Poisson with the given mean. Thus the simple Poisson is a special case of a mixed Poisson distribution.

As above, it can be shown that regardless of the choice of  $g(c)$ ,  $X$  will have a VTMR bigger than one, unless of course the above situation holds and  $g(c)$  puts all its mass at one value of  $c$  except in the special case where  $X$  is a simple Poisson random variable.

### THE NEGATIVE BINOMIAL DISTRIBUTION

The negative binomial distribution is both a mixed Poisson distribution, where the mixing distribution (often called the "prior" distribution) is a Gamma distribution, and a compound Poisson distribution where the compounding density  $g(n)$  is a logarithmic density,

$$g(n) = (1 - p)^n / (-n \times \ln(p)) .$$

Because of the mathematical convenience of these mixing and compounding distributions and the convenience of the negative binomial

<sup>2</sup>This simple proof is due to Gus Haggstrom (personal communication).

itself, it is the universal choice of logisticians when modeling failures as either a mixed or compound Poisson distribution.

It has the density

$$P(X = k) = C(k + r - 1, k)(1 - p)^k p^r$$

and the expectation and variance

$$E(X) = r(1 - p)/p$$

$$\text{Var}(X) = r(1 - p)/p^2$$

and hence the VTMR is

$$\text{VTMR} = 1/p ,$$

which is always greater than one, except (as mentioned above) in the limiting case where the VTMR is one and the distribution is simple Poisson.

## Appendix F

### MODELING VARIANCE-TO-MEAN RATIOS

Capability assessment and requirement models use VTMRs only indirectly, or not at all, as measures of the variability of arrival processes. Instead VTMRs are used as parameters describing the distribution of the number of items in the repair pipeline.<sup>1</sup> Despite this, VTMR estimation is done with demand data, and the estimates are of the VTMR of the demand process. When does the VTMR of the demand process become the VTMR of the pipeline process? Sufficient conditions that the VTMR of the pipeline be the VTMR of the demand process have received almost no attention in the literature other than the case of a simple Poisson demand process. This section describes several different conditions that assure the validity of this standard assumption. In particular, the same complete independence is required between the demand process and the repair process that is required by Palm's Theorem.<sup>2</sup> Other requirements are discussed below.

Although there is almost universal agreement that the repair process cannot, in general, be independent of the demand process, all VTMR estimation, other than in the above sections of this report, is concerned with demand data. If pipeline data were not available, this single-minded concern with the VTMR of the demand stream would be justified, but pipeline data are readily available as part of the D-143H system, and more recently part of the Weapon System Management Information System. Thus, there is little reason to continue estimating the VTMR of the demand process in an attempt to impute the VTMR of the pipeline for capability assessment models.<sup>3</sup>

Unfortunately, in peacetime, we cannot measure the VTMR of the wartime pipeline process. Estimating the VTMR of the wartime pipeline process is important and necessary and requires a giant leap of

<sup>1</sup>In particular this is true of RAND's Dyna-METRIC, AFLC's D041, LMF's Aircraft Availability Model, and the Standard Base Supply System fill rate formula. The only exception that comes to mind is the SRA model developed at PACAF/OA (Hiller, 1986; Hiller and Landis, 1986), which uses average backorders to estimate average pipeline quantities.

<sup>2</sup>In particular, complete independence seems to preclude the existence of queues, hence is often called the infinite server assumption. See Crawford, 1981, for a complete statement of the independence requirement.

<sup>3</sup>Obvious exceptions are studies to evaluate the effect that changes in the repair system will have on the distribution of the number of items in repair.

faith. However, the tacitly assumed argument that wartime pipeline VTMRs are more nearly approximated by peacetime demand process VTMRs than by peacetime pipeline VTMRs has yet to be formally considered, much less justified. Having said that, the following retreats from it and proceeds in the traditional manner with the assumption of independence between the demand process and the repair process. The goal is to prove additional forms of Palm's Theorem for nonhomogeneous Poisson processes giving sufficient conditions that the VTMR of the demand process is the VTMR of the pipeline process.

Virtually all capability assessment and requirements models draw on Palm's theorem (Palm, 1943) or generalizations of it. To the best of my knowledge the generalizations have not appeared in any of the common journals. Generalizations have appeared in: Crawford (1977); Hillestad and Carrillo (1980); Crawford (1981).

The most general and complete statement and proof of the theorem occur in Crawford (1981), which is repeatedly referenced here. However, any of the above articles contains an informative treatment.

Efforts to model the high degree of variation observed in demand and pipeline data usually assume the number of parts in the repair pipeline is a compound Poisson process or a mixed Poisson distribution. Following are several corollaries to the generalized form of Palm's theorem that give some legitimacy to the assumptions made in these models.

Suppose that item demands are a stationary simple Poisson process with parameter  $m$ , but  $m$  is unknown and randomly distributed according to a Gamma law with known parameters. Parzen (1957) has shown that for deterministic repair times the pipeline quantities are then negative binomial and have the same VTMR. The same result holds in the case of a mixture of nonhomogeneous Poisson processes, without restrictions on the repair distributions, other than requiring that they be measurable. Demands are assumed to arrive according to a nonstationary simple Poisson arrival process with mean value function  $m(t) = m \times h(t)$  where  $h(t)$  is nonstationary but known, and  $m$  is constant but unknown and has a Gamma distribution. For instance, this models the situation where the flying program varies in a known manner, and the demand rate per flying hour is constant but unknown.

Given the flexibility of the Gamma distribution, little modeling generality is lost in assuming  $m$  has that distribution.

### PALM'S THEOREM FOR MIXTURES OF NONSTATIONARY POISSON ARRIVAL PROCESSES

Theorem: Let  $X(t)$  be a Poisson arrival process such that  $E(X(t)) = m \times H(t)$ , where  $H(t)$  is known and  $m$  is unknown but has a Gamma distribution with a VTMR of  $\rho$ . Let the probability that an arrival at time  $s$  survives until time  $T$ ,  $s < T$ , be denoted  $\bar{F}(s, T)$ . Then, assuming independence of the demand process and the repair process, the number in the repair pipeline at time  $T$  is a negative binomial random variable with VTMR  $\rho$  and mean  $\mu$ ,

$$\mu = E(m) \int_s^T \bar{F}(u, T) h(u) du .$$

Proof: Conditioning on the value of  $m$ , it follows from the nonstationary form of Palm's theorem that given  $m = m'$ , the pipeline contents are simple Poisson with mean  $\mu'$ ,

$$\mu' = m' \int_s^T F(u, T) h(u) du ,$$

and thus the characteristic function of the number in the pipeline has the well-known form of the simple Poisson with multiplicative parameter  $m'$ . Unconditioning on the value of  $m$  yields the result.

### PALM'S THEOREM FOR CLUSTERED ARRIVALS WITH THE SAME REPAIR TIME

Theorem: Let  $X(t)$  be a compound Poisson arrival process with clustering density  $f$  and VTMR  $\rho$ . Under the above assumptions, and assuming that all arrivals in a cluster have the same repair time, then the number in repair at time  $T$  is a compound Poisson random variable with the same compounding density and VTMR as  $X(t)$  and mean  $\mu$ ,

$$\mu = \int_s^T F(u, T) h(u) du$$

Proof: It follows from the dynamic form of Palm's Theorem that the number of clusters in repair is a Poisson random variable, and since all repair times in a cluster are equal, the items in repair have the same compounding density, as was to be shown.

In particular it follows that if the arrival process is negative binomial, then the pipeline contents will be negative binomial with the same VTMR.

If all the items arriving in a given bunch do not have identical repair times, then in general the pipeline quantities will not be distributed as compound Poisson. However, the following theorem guarantees that the VTMR of the number in repair will be no larger than the VTMR of the arrival process.

### CLUSTERED ARRIVALS WITH DIFFERENT REPAIR TIMES

Theorem: Let  $X(t)$  be a compound Poisson arrival process,  $E(X(t)) = H(t)$ , and assume  $H(t)$  is the integral of its derivative  $h(t)$ . Under the above assumptions, if repair times are assigned randomly within a cluster, then the mean number  $\mu$  in repair at time  $T$  is given by the Palm's Theorem formula

$$\mu = \int_s^T F(u, T) h(u) du$$

and the VTMR of the number in repair is no greater than the VTMR of the arrival process.

Proof: Begin by assuming that the probability  $F(s, T)$ , that an arrival at  $s$  is still in repair at time  $T$ , is a simple function and is constant on intervals. Let  $S$  be such an interval and for  $s$  in this interval, let  $p = F(s, T)$ . Consider a  $Y(t)$  arrival process defined as identical to the  $X(t)$ , except that it includes only those arrivals that are still in repair at time  $T$ . The interarrival times between events in the  $Y$  process are the same as for the  $X$  process, only the clustering is different; and when restricted to  $S$ ,  $Y$  is also a compound Poisson process. The mean number of  $Y(t)$  arrivals in the interval  $S$  is given by

$$p[H(s_1, t) - H(s_1)] = \int_s^T F(u, T) h(u) du,$$

and, as mentioned above, the VTMR of the  $Y$  process is determined by its compounding density  $g$ . To evaluate the VTMR of the  $Y$  process, assume  $U$  to be the size of an  $X$  cluster, and  $V$  to be the size of the resulting  $Y$  cluster. Then,

$$E(V) = pE(U)$$

and,

$$\text{Var}(V) = E(\text{Var}(V | U)) + \text{Var}(E(V | U))$$



$$\begin{aligned}
&= E(p(1-p)U) + \text{Var}(pU) \\
&= p(1-p)E(U) + p^2\text{Var}(U) .
\end{aligned}$$

Using the above formula (see App. E ) for the VTMR of the process Y as a function of its cluster size and relating that to the VTMR of the X process:

$$\begin{aligned}
\text{VTMR}(Y) &= E(V) + \text{Var}(V) - E(V) \\
&= pE(U) + [p(1-p)E(U) + p^2\text{Var}(U)] - pE(U) \\
&= (1-p) + p[E(U) + \text{Var}(U) - E(U)] .
\end{aligned}$$

As  $p$  varies from 0 to 1, the latter quantity monotonically increases from 1 to the VTMR of the X process.

Continuing the above argument over all intervals  $S$ , the number in repair can be viewed as a sum of compound Poisson random variables with different compounding distributions whose VTMRs are never greater than the VTMR of the X process; and the mean number in repair is given by the sum of the above integrals, as was to be shown.

This proof glosses over the measure theoretic difficulties of showing that it suffices to consider functions  $F(s,t)$  that are constant on intervals. A formal proof of the adequacy of this class of functions could proceed as in Crawford (1981).

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Mathematical models of the logistics system are used to determine spares requirements and play an important role in evaluating logistics policies. The kernel of many, if not most, of these models is the modeling of the failure process and the resulting series of random demands on supply and maintenance. This report describes the assumptions of these models, and quantifies ways in which the behavior of the data differs from the assumptions of the models. The differences are pervasive and important. In addition, an examination of the number of parts in the repair pipeline over time reveals even more variability than does the number of demands over time. These observations have two important consequences: (1) excessive demand variability substantially reduces the confidence we can put in our requirements and capability assessment models; and (2) highly variable repair pipelines with means larger than assumed by requirements models have a damaging effect on aircraft availability and wartime readiness. Depot policies, decisions, and goals should be aimed at reducing these pipelines and increasing aircraft availability and wartime readiness.

*Keywords:*

*aircraft maintenance,  
F-15 aircraft, F-16 aircraft,  
C-5 aircraft*

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